11.2 Derivatives of Exponential Functions

Saturday, June 18, 2022 7:22 AM

$$y = e^{x}$$

$$y' = e^{x} \checkmark$$

$$= e^{x} (\ln e)(1)$$

$$y = a^{u(x)} \quad , \; a>0 \quad u(x) \text{ diff}$$

$$y' = a^{u(x)} (\ln a)(u'(x))$$

Exp Find y' if

① $y = e^{4x^3}$ $\Rightarrow y' = e^{4x^3} (\ln e)(12x^2)$

$$= 12x^2 e^{4x^3}$$

② $y = \underbrace{3x}_{\text{اول}} \, \underbrace{e^{3x^2+5x}}_{\text{ثاني}}$

$$y' = \underbrace{3x}_{\text{اول}} \underbrace{\left[e^{3x^2+5x} (\ln e)(6x+5) \right]}_{\text{مشتقة الثاني}} + e^{3x^2+5x} (3)$$

$$= 3x(6x+5)e^{3x^2+5x} + 3e^{3x^2+5x}$$

$$= 3e^{3x^2+5x}\left[x(6x+5)+1\right]$$

$$= 3e^{3x^2+5x}\left[x(6x+5)+1\right]$$

$$= 3e^{3x^2+5x}\left[6x^2+5x+1\right]$$

(3) $f(x) = 10^{2x+1}$ Find $f'(0)$

$$f'(x) = 10^{2x+1}(\ln 10)(2)$$

$$f'(0) = 10^{0+1}(\ln 10)(2) = 2(\ln 10)(10)$$

$$= 20 \ln 10$$

(4) $g(x) = \left(2e^{3x+1} - 5\right)^3$ Find g'

$$g'(x) = 3\left(2e^{3x+1} - 5\right)^2 \left[2e^{3x+1}(\ln e)(3) - 0\right]$$

$$= 18\left(2e^{3x+1} - 5\right)^2\left(e^{3x+1}\right)$$

$$= 18e^{3x+1}\left(2e^{3x+1} - 5\right)^2$$

$$g'(0) = 18e^{0+1}\left(2e^{0+1} - 5\right)^2$$

$$g'(0) = 18\, e \,(2e - 5)$$

$$= 18\, e\, (2e - 5)^2$$

(5) $h(x) = 3^{3x-6} + e^{x^2-4}$ Find $h'(2)$

$$h'(x) = 3^{3x-6}(\ln 3)(3) + e^{x^2-4}(\ln e)(2x)$$

$$h'(2) = 3^{6-6}(\ln 3)(3) + e^{4-4}(1)\,2(2)$$

$$= (1)(3)(\ln 3) + (1)(1)(4)$$

$$= \ln 3^3 + 4$$

$$= \ln 27 + 4$$

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(6) $y = \cancel{\ln} e^{x^2}$ Find $y''(1)$

$$y = x^2 (\ln e)$$

$$y = x^2 \Rightarrow y' = 2x \Rightarrow y'' = 2$$

$$y''(1) = 2$$

(7) $y = 4^x$

$$y' = y^x (\ln y)(1) = (\ln y)\, y^x$$

⑧ $y = x^4$ (poly)

$$y' = 4x^3$$ ✓

⑨ $y = 5^{x^2+x}$ Find $y'(0)$

$$y' = 5^{x^2+x} (\ln 5) (2x+1)$$

$$y'(0) = 5^{0+0} (\ln 5) (0+1)$$

$$= (1)(\ln 5)(1)$$

$$= \ln 5$$

⑩ $y = \underset{\text{poly}}{x^5} + \underset{\text{exp}}{5^x}$

$$y' = 5x^4 + 5^x (\ln 5)(1)$$

$$= 5x^4 + (\ln 5)\, x^5$$

(11) $$y = x^3 e^{x^2}$$

الأول   الثاني

pol   exp

$$y' = x^3 \left[e^{x^2} (\ln e)(2x)\right] + e^{x^2}\left[3x^2\right]$$

$$= 2x^4 e^{x^2} + 3x^2 e^{x^2}$$

$$= x^2 e^{x^2}\left[2x^2 + 3\right]$$

(12) $$y = 3^{x^2} \log_3 x^2$$

$$= 3^{x^2} \frac{\ln x^2}{\ln 3}$$

$$= \frac{1}{\ln 3}\, 3^{x^2}\, \ln x^2$$

الأول   الثاني

$$y' = \frac{1}{\ln 3}\, 3^{x^2}\, \frac{2x}{x^2} + \ln x^2 \,\frac{1}{\ln 3}\, 3^{x^2} (\ln 3)(2x)$$

$$y = \frac{}{\ln 3} \quad \frac{}{x^2} \quad \ln 3$$

$$= \frac{2}{\ln 3}\,\frac{3^{x^2}}{x} + 2\,3^{x^2} x \ln x^2$$

$$= 2\,3^{x^2}\left[\frac{1}{(\ln 3)\,x} + x \ln x^2\right]$$

Exp (Future Value)

Assume \$ 100 is invested at 8% compounded continuously so that the future value is

$$S(t) = 100\,e^{0.08t} \qquad t : \text{years}$$

1) At what rate the money in this account is growing at the end of one year

derivative

future value

$$S'(1) \Rightarrow S'(t) = 100\,e^{0.08t}\,(\ln e)\,(0.08)$$

$$S'(1) = \underline{100}\,e^{0.08}\,(1)\,\underline{\underline{(0.08)}}$$

0.08

$= 8\, e^{0.08}$

$= 8.666$ dollars

(b) At what rate the future value changes in the end of 10 years

$$S'(10) = 100\, e^{0.08(10)}\,(1)\,(0.08)$$

$$= 100\, e^{0.8}\,(0.08)$$

$$= 8\, e^{0.8}$$

$$\approx \$\, 17.804$$