

[4] [a] show that if $f(x)$ is even and $g(x)$ is odd then $(g \circ f)(x)$ is even

$$\begin{aligned} f(x) \text{ is even} &\Rightarrow f(-x) = f(x) \quad \forall x \\ g(x) \text{ is odd} &\Rightarrow g(-x) = -g(x) \quad \forall x \end{aligned}$$

$$\begin{aligned} (g \circ f)(-x) &= g(f(-x)) = g(f(x)) \quad \text{since } f \text{ is even} \\ &= (g \circ f)(x) \end{aligned}$$

[b] show that if $f(x)$ is even and $g(x)$ is odd then $\frac{f(x)}{g(x)}$ is odd

$$\frac{f(x)}{g(x)} = \left(\frac{f}{g}\right)(x)$$

$$\left(\frac{f}{g}\right)(-x) = \frac{f(-x)}{g(-x)}$$

$$= \frac{f(x)}{-g(x)} \quad \text{since } f \text{ even and } g \text{ odd}$$

$$= -\frac{f(x)}{g(x)}$$

$$= -\left(\frac{f}{g}\right)(x)$$

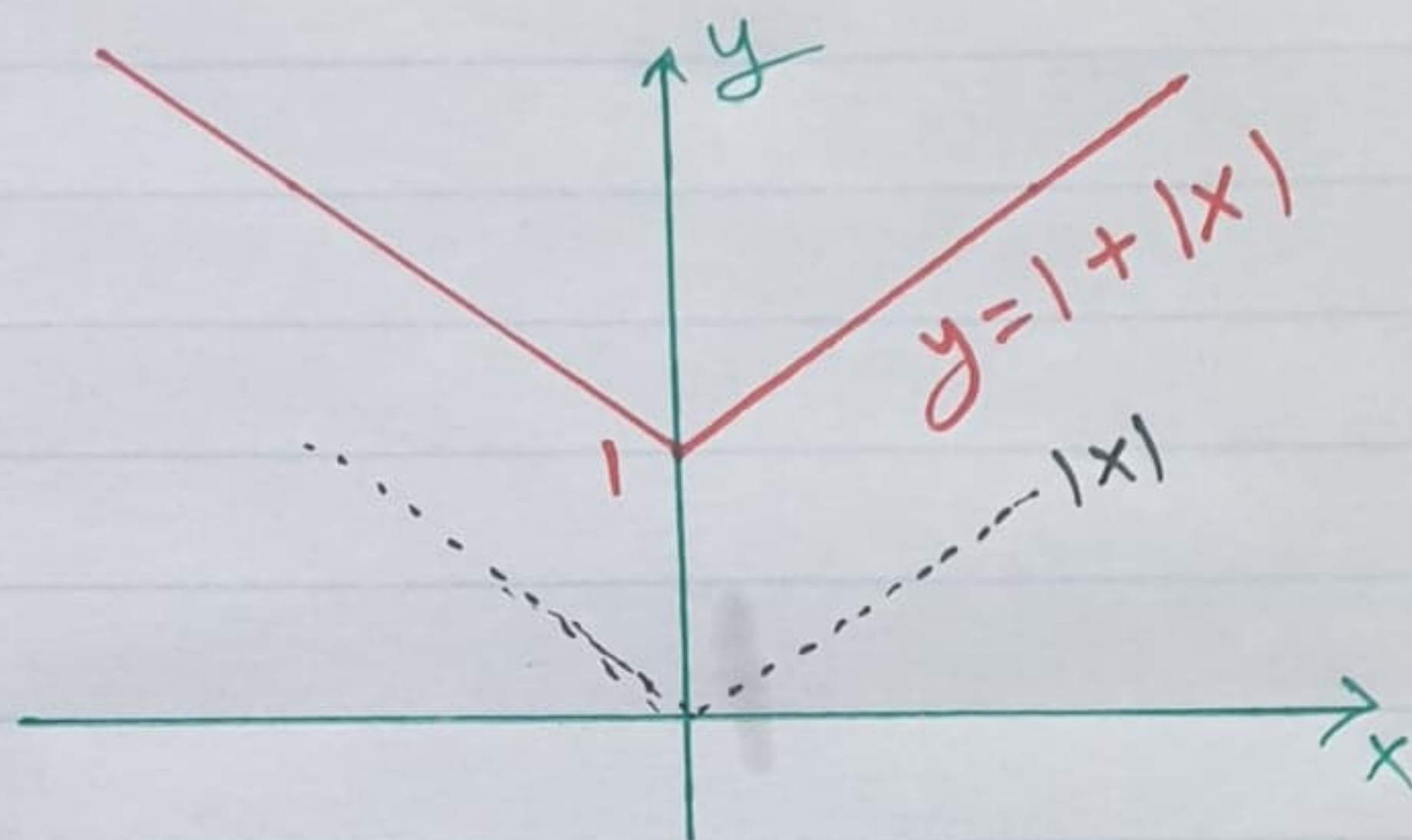
Note: • Even Functions are symmetric about y-axis
• Odd functions are symmetric about origin (0,0)

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1) [c] Find D and R of $f(x) = 1 + |x|$

$$D = \mathbb{R} = (-\infty, \infty)$$

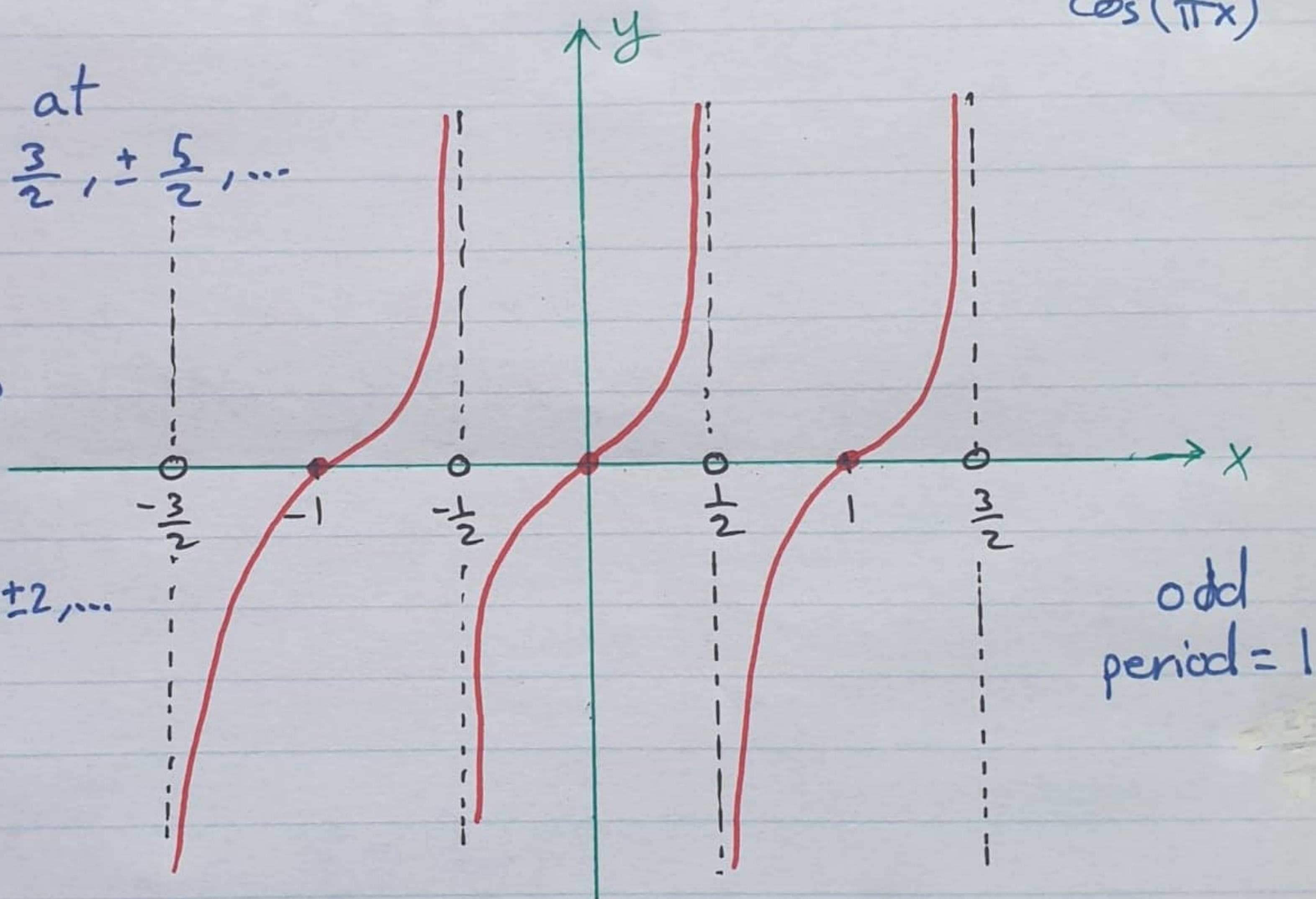
$$R = [1, \infty)$$



1) [b] Find D and R of $f(x) = \tan(\pi x) = \frac{\sin(\pi x)}{\cos(\pi x)}$

- $\cos(\pi x) = 0$ at $x = \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \dots$

- $\tan(\pi x) = 0$ when $\sin(\pi x) = 0$ at $x = 0, \pm 1, \pm 2, \dots$



$$\text{Domain} = \mathbb{R} \setminus \left\{ \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \dots \right\}$$

$$R = \mathbb{R}$$