

6.1

Simple Interest and Sequences

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The simple interest I is defined by $I = Prt$ where

- P is the principal or face value or present value
- r is the annual interest rate, $r \in [0, 1]$
- t is the time in years, $1 \text{ year} = 12 \text{ months} = 52 \text{ weeks}$

The future value S is defined by $S = P + I$ where

the principle P and the simple interest I are in dollars

Ex If \$8000 is invested for 2 years at annual interest rate of 9%

① how much interest will be received at the end of these 2 years?

$$P = \$8000, t = 2 \text{ years}, r = \frac{9}{100} = 0.09$$

$$I = Prt = (8000)(0.09)(2) = \$1440 \text{ is the simple interest}$$

② Find the future value for this investment.

$$S = P + I = 8000 + 1440 = \$9440$$

Ex If \$4000 is borrowed for 39 weeks at annual interest rate

15%, ① how much interest is due at the end of 39 week?

$$P = \$4000, t = \frac{39}{52} \text{ years}, r = \frac{15}{100} = 0.15$$

The simple interest is $I = Prt = (4000)(0.15)\left(\frac{39}{52}\right) = \450

② Find the future value of this loan

$$\begin{aligned} S &= P + I = 4000 + 450 \\ &= 4450 \text{ dollars} \end{aligned}$$

Exp If \$2000 is borrowed for one-half year at simple interest rate of 12% per year, what is the future value of the loan at the end of the half year?

$$P = \$2000, t = \frac{1}{2} \text{ years}, r = \frac{12}{100} = 0.12$$

$$\text{The simple interest is } I = Prt = (2000)(0.12)(\frac{1}{2}) = \$120$$

$$\text{The future value is } S = P + I = 2000 + 120 = 2120 \text{ dollars}$$

Exp An investor wants to have \$20,000 in 9 months. If the best interest rate is 6.05% per year. How much must be invested now to yield the desired amount?

$$P = ??, t = \frac{9}{12} \text{ years}, r = \frac{6.05}{100} = 0.0605, S = \$20,000$$

$$S = P + I$$

$$= P + Prt$$

$$20,000 = P + P(0.0605)(\frac{9}{12})$$

$$20,000 = P + 0.0454 P$$

$$20,000 = 1.0454 P$$

$$P = \frac{20,000}{1.0454} = \$19,131.4$$

is the present value
(Principal)

Exp If \$1000 is invested at 4% simple interest per year.

How long will it take to grow this amount to become \$1100?

$$P = \$1000, t = ??, r = \frac{4}{100} = 0.04, S = \$1100$$

$$S = P + I$$

$$= P + Prt$$

$$1100 = 1000 + (1000)(0.04)t$$

$$100 = 40t$$

$$t = \frac{100}{40} = 2.5 \text{ years}$$

Exp An investor wants to have \$ 1030 in 3 months.

If \$ 1000 is invested now, then what is the interest rate?

$$P = \$1000, t = \frac{3}{12} \text{ years}, r = ??, S = \$1030$$

$$S = P + I$$

$$= P + Prt$$

$$1030 = 1000 + (1000)(r)\left(\frac{3}{12}\right)$$

$$30 = 250 r$$

$$r = \frac{30}{250} = 0.12 = 12\%$$

A sequence is an order list of numbers $a_1, a_2, a_3, \dots, a_n, \dots$ where

a_1 : 1st term

a_2 : 2nd term

\vdots
 a_n : nth term

Exp write the first four terms of the sequence $a_n = \frac{(-1)^n}{2^n}$

$$1^{\text{st}} \text{ term is } a_1 = \frac{(-1)^1}{2(1)} = -\frac{1}{2}$$

$$2^{\text{nd}} \text{ term is } a_2 = \frac{(-1)^2}{2(2)} = \frac{1}{4}$$

$$3^{\text{rd}} \text{ term is } a_3 = \frac{(-1)^3}{2(3)} = -\frac{1}{6}$$

$$4^{\text{th}} \text{ term is } a_4 = \frac{(-1)^4}{2(4)} = \frac{1}{8}$$