Synchronous Motors Suggested Problems

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5-7. A 208-V Y-connected synchronous motor is drawing 50 A at unity power factor from a 208-V power system. The field current flowing under these conditions is 2.7 A. Its synchronous reactance is 1.6 Ω. Assume a linear open-circuit characteristic.

(a) Find $\mathbf{V}_\phi$ and $\mathbf{E}_A$ for these conditions.

(b) Find the torque angle δ.

(c) What is the static stability power limit under these conditions?

(d) How much field current would be required to make the motor operate at 0.80 PF leading?

(e) What is the new torque angle in part *(d)*?

SOLUTION

(a) The phase voltage of this motor is V_ϕ = 120 V, and the armature current is $\mathbf{I}_A = 50\angle 0° \text{ A}$. Therefore, the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi - R_A\mathbf{I}_A - jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = 120\angle 0° \text{ V} - j(1.6\ \Omega)(50\angle 0° \text{ A})$$

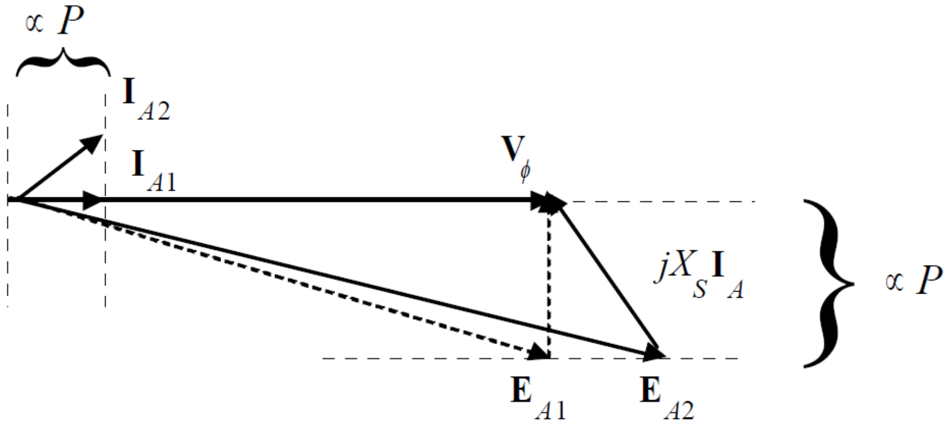
$$\mathbf{E}_A = 144\angle -33.7° \text{ V}$$

(b) The torque angle δ of this machine is –33.7°.

(c) The static stability power limit is given by

$$P_{\max} = \frac{3V_\phi E_A}{X_S} = \frac{3(120\text{ V})(144\text{ V})}{(1.6\ \Omega)} = 32.4\text{ kW}$$

(d) A phasor diagram of the motor operating at a power factor of 0.80 leading is shown below.



Since the power supplied by the motor is constant, the quantity $I_A \cos\theta$, which is directly proportional to power, must be constant. Therefore,

$$I_{A2}(0.8) = (50\text{ A})(1.00)$$

$$\mathbf{I}_{A2} = 62.5\angle 36.87^\circ\text{ A}$$

The internal generated voltage required to produce this current would be

$$\mathbf{E}_{A2} = \mathbf{V}_\phi - R_A\mathbf{I}_{A2} - jX_S\mathbf{I}_{A2}$$

$$\mathbf{E}_{A2} = 120\angle 0^\circ\text{ V} - j(1.6\ \Omega)(62.50\angle 36.87^\circ\text{ A})$$

$$\mathbf{E}_{A2} = 197\angle -23.9^\circ\text{ V}$$

The internal generated voltage E_A is directly proportional to the field flux, and we have assumed in this problem that the flux is directly proportional to the field current. Therefore, the required field current is

$$I_{F2} = \frac{E_{A2}}{E_{A1}} I_{F1} = \frac{197 \text{ V}}{144 \text{ V}} (2.7 \text{ A}) = 3.70 \text{ A}$$

(e) The new torque angle δ of this machine is –23.9°.

5-13. A 480-V 100-kW 0.8-PF leading 50-Hz four-pole Y-connected synchronous motor has a synchronous reactance of 1.8 Ω and a negligible armature resistance. The rotational losses are also to be ignored. This motor is to be operated over a continuous range of speeds from 300 to 1500 r/min, where the speed changes are to be accomplished by controlling the system frequency with a solid-state drive.

(a) Over what range must the input frequency be varied to provide this speed control range?

(b) How large is E_A at the motor's rated conditions?

(c) What is the maximum power the motor can produce at rated speed with the E_A calculated in part *(b)*?

(d) What is the largest value that E_A could be at 300 r/min?

(e) Assuming that the applied voltage V_ϕ is derated by the same amount as E_A, what is the maximum power the motor could supply at 300 r/min?

(f) How does the power capability of a synchronous motor relate to its speed?

SOLUTION

(a) A speed of 300 r/min corresponds to a frequency of

$$f_{se} = \frac{n_m P}{120} = \frac{(300 \text{ r/min})(4)}{120} = 10 \text{ Hz}$$

A speed of 1500 r/min corresponds to a frequency of

$$f_{se} = \frac{n_m P}{120} = \frac{(1500 \text{ r/min})(4)}{120} = 50 \text{ Hz}$$

The frequency must be controlled in the range 10 to 50 Hz.

(b) The armature current at rated conditions is

$$I_A = I_L = \frac{P}{\sqrt{3}\ V_T\ \text{PF}} = \frac{100\text{ kW}}{\sqrt{3}(480\text{ V})(0.8)} = 150.3\text{ A}$$

so $\mathbf{I}_A = 150.3\angle 36.87°\text{ A}$. This machine is Y-connected, so the phase voltage is $V_\phi = 480/\sqrt{3} = 277\text{ V}$. The internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi - R_A\mathbf{I}_A - jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = 277\angle 0°\text{ V} - j(1.8\ \Omega)(150.3\angle 36.87°\text{ A})$$

$$\mathbf{E}_A = 489\angle -26.2°\text{ V}$$

So $E_A = 489\text{ V}$ at rated conditions.

(c) The maximum power that the motor can produce at rated speed with the value of E_A from part *(b)* is

$$P_{\max} = \frac{3V_\phi E_A}{X_S} = \frac{3(277\text{ V})(489\text{ V})}{1.8\ \Omega} = 226\text{ kW}$$

(d) Since E_A must be decreased linearly with frequency, the maximum value at 300 r/min would be

$$E_{A,300} = \left(\frac{10\text{ Hz}}{50\text{ Hz}}\right)(489\text{ V}) = 97.8\text{ V}$$

(e) If the applied voltage V_ϕ is derated by the same amount as E_A, then V_ϕ = (10/50)(277) = 55.4 V. Also, note that X_S = (10/50)(1.8 Ω) = 0.36 Ω. The maximum power that the motor could supply would be

$$P_{\max} = \frac{3V_\phi E_A}{X_S} = \frac{3(97.8 \text{ V})(55.4 \text{ V})}{0.36\ \Omega} = 45.1 \text{ kW}$$

(f) As we can see by comparing the results of *(c)* and *(e)*, the power-handling capability of the synchronous motor varies *linearly* with the speed of the motor.

5-17. A 440-V, 60 Hz, three-phase Y-connected synchronous motor has a synchronous reactance of 1.5 Ω per phase. The field current has been adjusted so that the torque angle δ is 25° when the power supplied by the generator is **80** kW.

(a) What is the magnitude of the internal generated voltage $\mathbf{E}_A$ in this machine?

(b) What are the magnitude and angle of the armature current in the machine? What is the motor's power factor?

(c) If the field current remains constant, what is the absolute maximum power this motor could supply?

SOLUTION

(a) The power supplied to the motor is 80 kW. This power is given by the equation

$$P = \frac{3V_\phi E_A}{X_S}\sin\delta$$

The phase voltage of the generator is $440/\sqrt{3} = 254\text{ V}$, so the magnitude of $\mathbf{E}_A$ is

$$E_A = \frac{X_S\,P}{3V_\phi \sin\delta} = \frac{(1.5\ \Omega)(80\text{ kW})}{3(254\text{ V})\sin 25^\circ} = 373\text{ V}$$

(b) The armature current in this machine is given by

$$\mathbf{I}_A = \frac{\mathbf{V}_\phi - \mathbf{E}_A}{jX_S} = \frac{254\angle 0° \text{ V} - 373\angle -25°}{j1.5} = 119\angle 28° \text{ A}$$

The power factor of the motor is PF = cos 28º = 0.883 leading.

(c) The maximum power that the motor could supply at this field current is

$$P_{\max} = \frac{3V_\phi E_A}{X_S} = \frac{3(254 \text{ V})(373 \text{ V})}{1.5\ \Omega} = 189.4 \text{ kW}$$

5-19. A 100-hp 440-V 0.8-PF-leading Δ-connected synchronous motor has an armature resistance of 0.3 Ω and a synchronous reactance of 4.0 Ω. Its efficiency at full load is 96 percent.

(a) What is the input power to the motor at rated conditions?

(b) What is the line current of the motor at rated conditions? What is the phase current of the motor at rated conditions?

(c) What is the reactive power consumed by or supplied by the motor at rated conditions?

(d) What is the internal generated voltage $\mathbf{E}_A$ of this motor at rated conditions?

(e) What are the stator copper losses in the motor at rated conditions?

(f) What is P_{conv} at rated conditions?

(g) If E_A is decreased by 10 percent, how much reactive power will be consumed by or supplied by the motor?

Solution

(a) The input power to the motor at rated conditions is

$$P_{\text{IN}} = \frac{P_{\text{OUT}}}{\eta} = \frac{(100\text{ hp})(746\text{ W/hp})}{0.96} = 77.7\text{ kW}$$

(b) The line current to the motor at rated conditions is

$$I_L = \frac{P}{\sqrt{3}\,V_T\text{ PF}} = \frac{77.7\text{ kW}}{\sqrt{3}(440\text{ V})(0.85)} = 120\text{ A}$$

The phase current to the motor at rated conditions is

$$I_\phi = \frac{I_L}{\sqrt{3}} = \frac{120\text{ A}}{\sqrt{3}} = 69.3\text{ A}$$

(c) The reactive power supplied by this motor to the power system at rated conditions is

$$Q_{\text{rated}} = 3V_\phi I_A \sin\theta = 3(440\text{ V})(69.3\text{ A})\sin 36.87° = 54.9\text{ kVAR}$$

(d) The internal generated voltage at rated conditions is

$$\mathbf{E}_A = \mathbf{V}_\phi - R_A\mathbf{I}_A - jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = 440\angle 0°\text{ V} - (0.3\ \Omega)(69.3\angle 36.87°\text{ A}) - j(4.0\ \Omega)(69.3\angle 36.87°\text{ A})$$

$$\mathbf{E}_A = 634\angle -21.7°\text{ V}$$

(e) The stator copper losses at rated conditions are

$$P_{\text{CU}} = 3I_A^{\ 2}R_A = 3(69.3\text{ A})^2(0.3\ \Omega) = 4.3\text{ kW}$$

(f) P_{conv} at rated conditions is

$$P_{\text{conv}} = P_{\text{IN}} - P_{\text{CU}} = 77.7 \text{ kW} - 4.3 \text{ kW} = 73.4 \text{ kW}$$

(g) If E_A is decreased by 10%, the new value if E_A = (0.9)(634 V) = 571 V. To simplify this part of the problem, we will ignore R_A. Then the quantity $E_A \sin\delta$ will be constant as E_A changes. Therefore,

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_1\right) = \sin^{-1}\left(\frac{634 \text{ V}}{571 \text{ V}}\sin(-21.7°)\right) = -24.2°$$

Therefore,

$$\mathbf{I}_A = \frac{\mathbf{V}_\phi - \mathbf{E}_A}{jX_S} = \frac{440\angle 0° \text{ V} - 571\angle -24.2°}{j4.0} = 61.9\angle 19° \text{ A}$$

and the reactive power supplied by the motor to the power system will be

$$Q = 3V_\phi I_A \sin\theta = 3(440 \text{ V})(61.9 \text{ A})\sin(19°) = 26.6 \text{ kVAR}$$

Many Thanks
for
Your Attention!

Reference

- Instructor's Solutions Manual to accompany Electric Machinery Fundamentals by Stephen Chapman, 5^{th} Ed., McGraw-Hill, Inc., 2012.