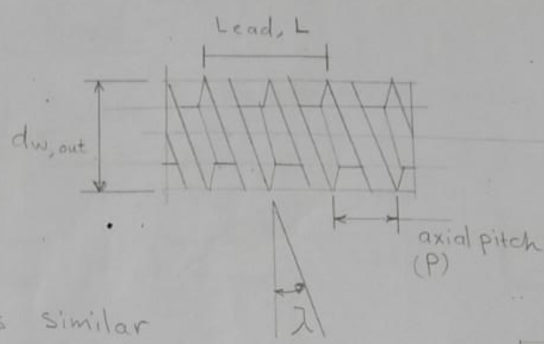
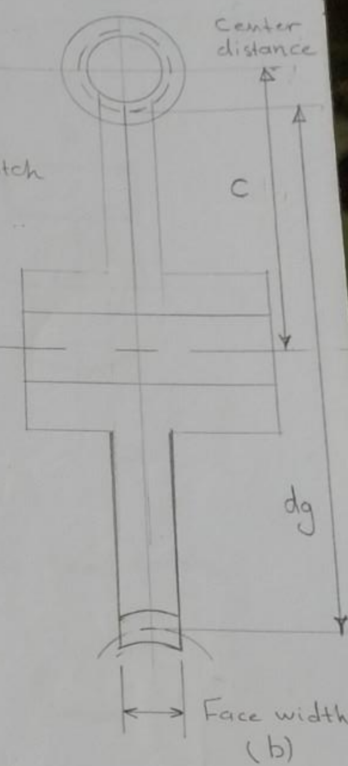
Worm Gear:





Geometry of Worm is similar to power screw.

Geometry of worm gear is similar to helical gear, but the teeth are curved to envelop the worm.

Rotation of worm gear simulate involute rack.

worm gear lead angle (λ) [correspond to screw lead angle].

Gear helix angle (ψ)

$\lambda = \psi$ and must be of the same hand.

pitch dia. of worm gear:

$$d_g = \frac{N_g \, P}{\pi}, \quad P = \text{circular pitch of gear.}$$

pitch dia. of worm is not a function of no. of teeth N_w

$\Rightarrow$ Velocity ratio cannot determined by ratio of diameters d_w, d_g.

$$\Rightarrow \frac{\omega_w}{\omega_g} = \frac{N_g}{N_w}$$

$N_g > 24$, $N_w + N_g > 40$

worm of any pitch dia. can be made with any N_w and any axial pitch [d_w is not function of N_w, or L].

For max. power transmission:

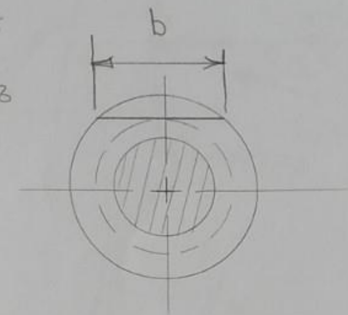
Recommended value $= \left(\dfrac{C^{0.875}}{2.2}\right)$

$$\frac{C^{0.875}}{3.0} \leq d_w \leq \frac{C^{0.875}}{1.7}$$

C = mm

$$\frac{C^{0.875}}{2.002} \leq d_w \leq \frac{C^{0.875}}{1.068}$$

Face width of worm gear:



$$b \leq 0.5\,(d_w)_{out}$$

Lead angle of worm:

$$\tan\lambda = \frac{L}{\pi d_w}$$

- To avoid interference → pressure angle is related to lead angle.

Standard axial pitch of worm [circular pitch of gear]:

$\frac{1}{4}$, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{1}{2}$, 2 inch

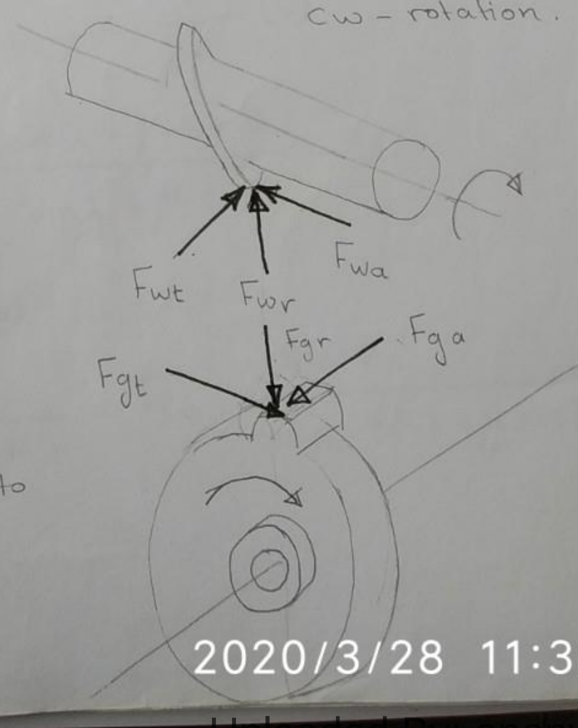
For shaft angle, $\Sigma = 90°$, $(P_{axial})_W = (P_{circular})_G$, $P_a = \frac{L}{N_w}$, $P_c = \frac{\pi d_g}{N_g}$

Table [13-9] – Max. lead angle (λ) for various pressure angles (ϕ_n).

Worm Gear force analysis:

- For worm gear set of $\Sigma = 90°$

RH- worm
Cw - rotation.



$$F_{w_t} = F_{g_a}$$

$$F_{g_t} = F_{w_a}$$

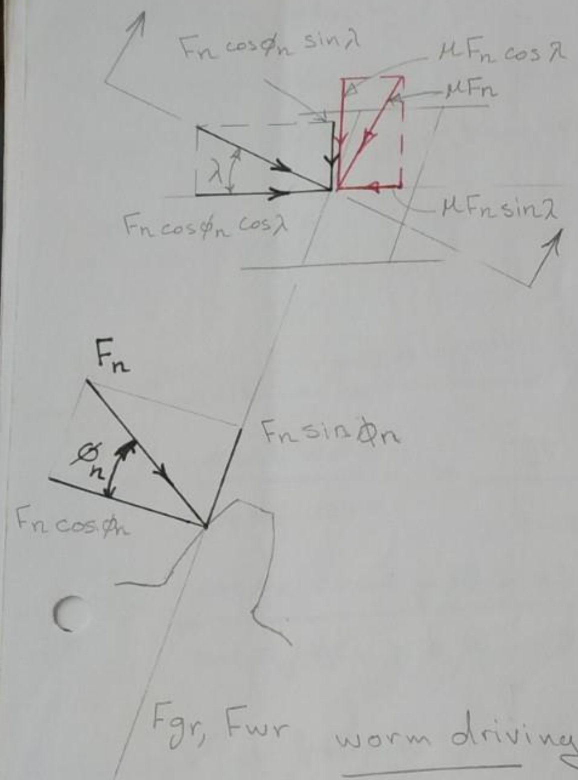
$$F_{w_r} = F_{g_r}$$

Normally:

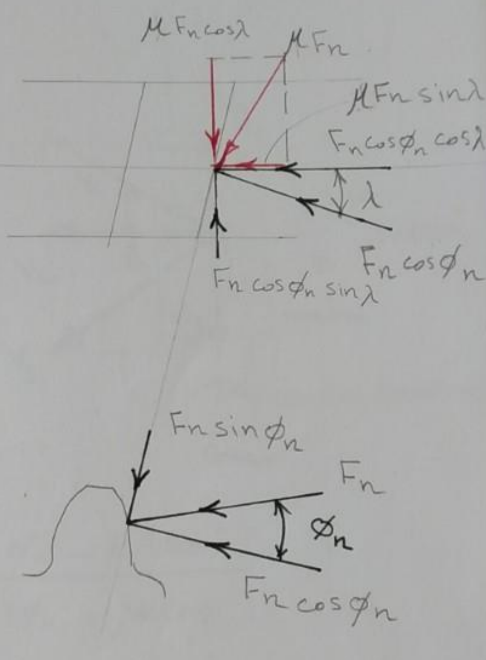
Driver = Worm, driven = gear.

The worm force analysis is analogus to power screws force analysis.

where $\alpha = \phi_n$



worm driving



gear driving

Driving worm:

cw rotation of worm.

$$F_{gt} = F_{wa} = F_n \cos\phi_n \cos\lambda - \mu F_n \sin\lambda \quad \text{——(a)}$$

$$F_{wt} = F_{ga} = F_n \cos\phi_n \sin\lambda + \mu F_n \cos\lambda \quad \text{——(b)}$$

$$F_{gr} = F_{wr} = F_n \sin\phi_n \quad \text{——(c)}$$

From (a), (b) :
$$\frac{F_{gt}}{F_{wt}} = \frac{\cos\phi_n \cos\lambda - \mu \sin\lambda}{\cos\phi_n \sin\lambda + \mu \cos\lambda} \quad \text{——(d)}$$

combining (a) and (c) and (c), (b)

$$F_{gr} = F_{wr} = F_{gt} \frac{\sin\phi_n}{\cos\phi_n \cos\lambda - \mu \sin\lambda} = F_{wt} \frac{\sin\phi_n}{\cos\phi_n \sin\lambda + \mu \cos\lambda}$$

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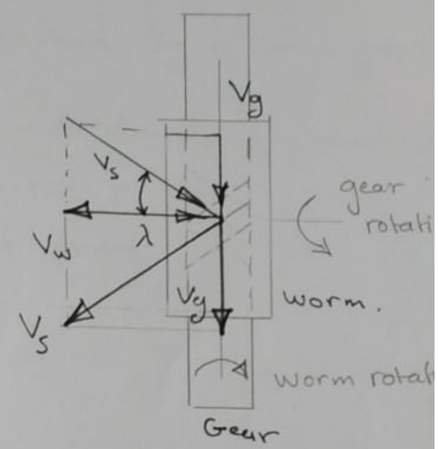
V_s = Sliding velocity

V_w = worm velocity (tangential)

V_g = gear velocity. (tangential)

$$\frac{V_g}{V_w} = \tan\lambda \quad , \quad V_s = \frac{V_w}{\cos\lambda} = \frac{V_g}{\sin\lambda}$$



Efficiency of worm gear:

$$e = \frac{\text{output work}}{\text{input work}} = \frac{F_{gt}\, V_g}{F_{wt}\, V_w}$$

$$\Rightarrow e = \frac{\cos\phi_n \cos\lambda - \mu \sin\lambda}{\cos\phi_n \sin\lambda + \mu\cos\lambda} \tan\lambda = \frac{\cos\phi_n - \mu\tan\lambda}{\cos\phi_n + \mu\cot\lambda}$$

Coefficient of friction (μ) depends on variables such as:

Sliding velocity, lubricant, temprature, Surface finish, material.

Fig. (13-42) coefficient of friction as a function of V_s (ft/min).

curve B = high quality worm gear [case hardened worm – phosphor bronze gear]

Curve A = more friction is expected [cast iron worm and worm gear].

Equation (a) if μ is sufficiently high $\rightarrow F_{tg} \rightarrow 0$

$\Rightarrow$ Self-locking gear, [occurs for gear driving] $\rightarrow T_w = 0$

Self-locking desirable to hold the load from reversing.

Gear Driving

F_n is reversed as for worm driving.

Since sliding velocity has the same direction as for worm driving

$\Rightarrow$ Friction has the same direction.

$$\Rightarrow F_{tw} = F_n \cos\phi_n \sin\lambda - \mu F_n \cos\lambda$$

For self-locking : $\mu \geqslant \cos\phi_n \tan\lambda$

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Example:

Motor: 2 hp, 1200 rpm, drives a 60 rpm machine by a worm gear.

$C = 5''$; worm: RH, $P_a = \frac{5}{8}''$, $\phi_n = 14\frac{1}{2}°$, hardened and ground steel

gear: bronze. Double-thread, $N_w = 2$.

a) Force component corresponding to rated motor power.

b) power delived to driven machine

c) Drive self-locking or not.

Solution:

1- $N_w + N_g > 40$

$$\frac{\omega_w}{\omega_g} = \frac{1200}{60} = 20 = \frac{N_g}{N_w} \Rightarrow N_g = 20 N_w$$

Double threaded worm: $N_w = 2$

$\Rightarrow N_g = 40$

2- $P = \frac{5}{8}$, $d_g = \frac{P N_g}{\pi} = \frac{\frac{5}{8} \times 40}{\pi} = 7.96''$

3- $C = \frac{d_g + d_w}{2}$, $C = 5 \Rightarrow d_w = 2.04''$

4- Lead angle: $\lambda = \tan^{-1} \frac{L}{\pi d_w} = \tan^{-1} \frac{1.25}{\pi (2.04)} = 11.04°$

5- $V_w = \frac{\pi d_w n_w}{12} = \frac{\pi (2.04)\, 1200}{12} = 640$ fpm

6- $F_{wt} = F_{ga} = \frac{hp(33000)}{V_w} = \frac{2 \times 33000}{640} = 103$ lb.

7- $V_s = \frac{V_w}{\cos\lambda} = \frac{640}{\cos(11.04)} = 652$ fpm

From Fig. [13-42] curve (B) [bronz gear - hardened and ground worm]

$\mu = 0.026$

8- $$\frac{F_{gt}}{F_{wt}} = \frac{\cos\phi_n \cos\lambda - \mu \sin\lambda}{\cos\phi_n \sin\lambda + \mu \cos\lambda} = \frac{\cos(14.5)\cos(11.04) - 0.026 \sin(11.04)}{\cos(14.5) \sin(11.04) + 0.026 \cos(11.04)} = 4.48$$

$F_{gt} = F_{wa} = 103 (4.48) = 461$ lb

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9- $F_{gr} = F_{wr} = F_{gt} \dfrac{\sin \phi_n}{\cos\phi_n \cos\lambda - \mu \sin\lambda} = F_{wt} \dfrac{\sin\phi_n}{\cos\phi_n \sin\lambda + \mu\cos\lambda}$

$\Rightarrow F_{gr} = F_{wr} = 103 \dfrac{\sin(14.5)}{\cos(14.5)\sin(11.04) + 0.026\cos(11.04)} = 122 \text{ lb}$

10- $e = \dfrac{\cos\phi_n - \mu\tan\lambda}{\cos\phi_n + \mu\cot\lambda} = \dfrac{\cos(14.5) - 0.026\tan(11.04)}{\cos(14.5) + 0.026\cot(11.04)} = 87\%$

or $e = \dfrac{F_{gt} V_g}{F_{wt} V_w} = \dfrac{F_{gt}}{F_{wt}} \tan\lambda = 4.48 \tan 11.04 = 87\%$

11- To account for friction losses on bearing assume an overall efficiency, $e = 85\%$

$\Rightarrow$ output power = Input power $\times e = 2(0.85) = 1.7$ hp

12- for self locking

$\mu \geq \cos\phi_n \tan\lambda$

$\cos(14.5)\tan(11.04) = 0.19$, but $\mu = 0.026$

$\Rightarrow \mu < \cos\phi_n \tan\lambda \Rightarrow$ not self-locking drive.

Gear Driver:

$e_g = \dfrac{\cos\phi_n - \mu\cot\lambda}{\cos\phi_n + \mu\tan\lambda}$

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Allowable tangential force on worm gear:

$$(F_{gt})_{all} = C_s \, d_g^{0.8} \, b \, C_m C_v.$$

C_s = material factor.

d_g = mean gear dia. [mm] in

b = face width such that $b < 0.67 \, d_w$ [mm] in d_w = mean worm dia.

C_m = ratio correction factor.

C_v = velocity factor.

$$C_s = 270 + 10.37 \, C^3 \quad , \; C \leq 3''$$

Sand cast gear:

$$C_s = \begin{cases} 1000 & C>3 \quad d_g \leq 2.5'' \\ 1190 - 477 \log(d_g) & C>3 \, , \; d_g > 2.5'' \end{cases}$$

Chilled cast gears:

$$C_s = \begin{cases} 1000 & C>3 \quad d_g \leq 8'' \\ 1412 - 456 \log[d_g] & , C>3, \; d_g > 8'' \end{cases}$$

Centrifugally cast gears:

$$C_s = \begin{cases} 1000 & C>3 \quad d_g \leq 25'' \\ 1251 - 180 \log(d_g) & d_g > 25'' \end{cases}$$

C_m = ratio correction factor.

$$C_m = \begin{cases} 0.02\sqrt{-m_g^2 + 40m_g - 76} + 0.46 & 3 < m_g \leq 20 \\ 0.0107\sqrt{-m_g^2 + 56m_g + 5145} & 20 < m_g \leq 76 \\ 1.1483 - 0.00658\, m_g & m_g > 76 \end{cases}$$

C_v = velocity factor:-

$$C_v = \begin{cases} 0.659\, e^{(-0.0011 V_s)} & V_s < 700 \text{ fpm.} \\ 13.31\, V_s^{-0.571} & 700 \leq V_s < 3000 \\ 65.52\, V_s^{-0.774} & V_s > 3000 \text{ fpm.} \end{cases}$$

$$f = \begin{cases} 0.15, & V_s = 0 \\ 0.124\, e^{(-0.074 V_s^{0.645})}, & 0 < V_s < 10 \text{ fpm.} \\ 0.103\, e^{(-0.11 V_s^{0.45})} + 0.012, & V_s > 10 \text{ fpm.} \end{cases}$$

Worm:

$d_o = d_w + 2a$, $d_r = d - 2b$

max. Face width of worm

$$(F_w)_{max} = 2\sqrt{\left(\frac{D_t}{2}\right)^2 - \left(\frac{D}{2} - a\right)^2}$$

worm gear:

$D_t = d_g + 2a$, $D_r = d_g - 2b$

$a, b \rightarrow$ Table [15-8]

$a = \frac{P_a}{\pi}$, $b = \frac{1.157\, P_a}{\pi}$

clearance $\Rightarrow$ $c = b - a$

worm gear face width:

$$F = \begin{cases} \frac{2\, d_m}{3}, & P_a > 0.16 \text{ in } [4 \text{ mm}] \\ 1.125\sqrt{(d_o + 2c)^2 - (d_o - 4a)^2}, & P \leq 0.16 \text{ in } [4 \text{ mm}] \end{cases}$$

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Tangential force of gears:

$$F_g^t = \frac{H_o \, 33000}{V_g} \, n \, K_a$$

$H_o = e \, H_i$

n = design factor , K_a = application factor.

For Safe Design:

$$F_g t < (F_g t)_{all}$$

worm gear wear:

$$(F_g^t)_{all} = K_w \, d_g \, b$$

K_w = worm gear load factor. → Table [15 – 11]
d_g = gear pitch dia.
b = worm - gear face width.

⇒ for the gear to be safe in wear.

$$F_g t < (F_g t)_{all}$$

Bending stress

$$\sigma_b = \frac{W_g^t}{P_n \, b \, y}$$

$P_n = P \cos \lambda$ y = lewis form factor

$\phi_n = 14\frac{1}{2}°$ $y = 0.1$

$\phi_n = 20°$ $y = 0.125$

$\phi_n = 25°$ $y = 0.15$

$\phi_n = 30°$ $y = 0.175$

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Surface contact strength:

Table [15-4] → S_c [contact strength of worm gear]

$$\Rightarrow \quad n_c = \frac{S_c}{\sigma_H}$$

Worm Gear thermal Capacity

Worm gear rated capacity is limited by the heat generated, and the ability of housing to dissipate friction heat, of gear and lubricant temp.

Normally: $T < 200°F\ (93°C)$ → Satisfactory operation.

Rate of Heat dissipation:

$$H_T = CA(t_o - t_a).$$

H_T = rate of heat dissipaction ($\frac{ft\ lb}{min}$), ($\frac{w}{}$)

C_t = heat transfere coefficient [$\frac{ft \cdot lb}{min \cdot ft^2 \cdot F°}$]

where [ft^2 – surface area of housing.

A = Housing External area.

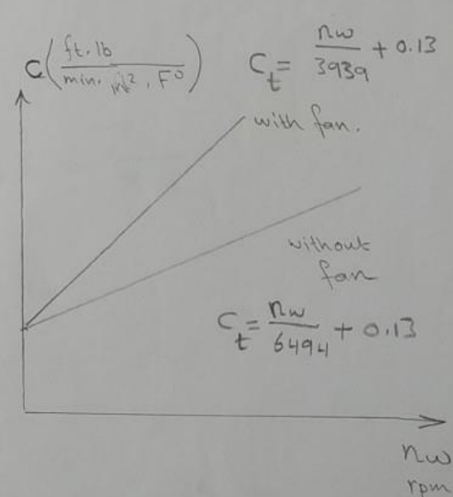
t_o = oil temp. [allowable temp., $T_o = 160 - 200\ F°$]

t_a = ambient air temp.

For conventional housing design:

$$A = 0.3\, C^{1.7}, \quad \text{rough estimation.}$$

$A = ft^2$, C = inches. , $A = 43.2\, C^{1.7}$ $A = Inch^2$



Greater housing area is obtained using cooling fins.

A rough estimation of heat transfere coefficient, C → Fig. [Notes]

power lost due to tooth friction:

$$H_f = \frac{F_f V_S}{33{,}000}\ (hp), \quad H_f = V_s F_f\ (Watt)$$

$$F_f = \mu F_n = \frac{\mu\ F_{gt}}{\cos\phi_n \cos\lambda - \mu \sin\lambda}$$

$$e = \frac{H_o}{H_i} = \frac{H_i - H_f}{H_i}$$

$$H_f = (1-e)\, H_i$$

H_f = power lost by friction = Heat dissipated = H_T

Example:

A 11:1 worm gear reducer.
worm: hardened steel
gear: chill-cast bronze.
Center distance, $C \cong 6$ in — approximatly
$W_w = 1200$ rpm,
Determine: $d_w, d_g, N_w, N_g, p, \lambda, \phi_n$
horse power, and efficiency.

Solution:

$$\frac{W_g}{W_w} = \frac{N_w}{N_g} \Rightarrow \frac{N_w}{N_g} = 11$$

If we choose, $N_w = 4$, $N_w + N_g > 40 \Rightarrow N_g = 44$

2- $$e = \frac{\cos\phi_n - \mu \tan\lambda}{\cos\phi_n + \mu \cot\lambda} \Rightarrow \lambda\uparrow \rightarrow e\uparrow$$ [λ as large as possible]

From table [13-7] : $\lambda\uparrow \rightarrow e\uparrow$
take max. possible λ.

3- $$\frac{C^{0.875}}{3.0} \leq d_w \leq \frac{C^{0.875}}{1.7}$$

For high $\lambda \rightarrow d_w$ must be small, $\tan\lambda = \frac{L}{\pi d_w}$

$$\Rightarrow \text{choose } d_w = \frac{C^{0.875}}{3.0} = \frac{(6)^{0.875}}{3.0} = 1.6 \text{ in}$$

$$C = \frac{d_w + d_g}{2} \Rightarrow d_g = 10.4 \text{ in}$$

$$P = \frac{\pi d_g}{N_g} = \frac{\pi(10.4)}{44} = 0.745 \Rightarrow \text{Standard } P = 0.75 \text{ in.}$$

Standard $[P_c, P_a] = \frac{1}{4}, \frac{5}{16}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{1}{2}, 2$

- slight increase in $P \rightarrow$ a larger gear or small worm < recommended
or increase (c).

Take $$d_g = \frac{N_g\ P}{\pi} = \frac{44(0.75)}{\pi} = 10.5 \text{ in}, \quad d_w = 1.6'' \Rightarrow C = 6.05$$

If we choose: $C = 6\frac{1}{8}''$, $d_g = 10.5''$ $\Rightarrow d_w = 1.75''$

$$(d_w)_{min} = \frac{C^{0.875}}{3.0} = \frac{(6.125)^{0.875}}{3.0} = 1.63 \text{ in}$$

$$\Rightarrow d_w > (d_w)_{min}$$

$$\Rightarrow d_g = 10.5 \text{ in} , \; d_w = 1.75 \text{ in} , \; C = 6\tfrac{1}{8}''$$

5- $\tan\lambda = \frac{L}{\pi d_w}$, $L = p N_w$ $\Rightarrow \tan\lambda = \frac{4(0.75)}{\pi(1.75)} \Rightarrow \lambda = 28.62°$

6- To find $e = \frac{\cos\phi_n - \mu\tan\lambda}{\cos\phi_n + \mu\cot\lambda}$ $\begin{cases} \text{From table [15-9]}, \lambda_{max} = 35° \\ \rightarrow \phi_n = 25° \end{cases}$

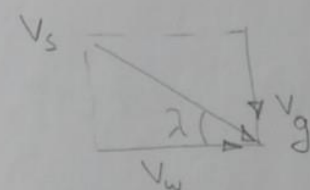
$$V_g = \frac{\pi d_g n_g}{12} = \pi\left(\frac{10.5}{12}\right) 1200 \left(\frac{1}{11}\right) = 300 \text{ ft/min}$$

$$V_w = \frac{\pi d_w n_w}{12} = \frac{\pi(1.75)\,1200}{12} = 550 \text{ ft/min}$$

Note: $V_g \neq V_w$ [because of sliding, not rolling].

To find μ — $V_s = \frac{V_w}{\cos\lambda} = \frac{V_g}{\sin\lambda} = \frac{300}{\sin 28.62}$

$$V_s = 626 \text{ fpm}.$$



From Fig. [13-42], Curve B [hardened worm - phosphor, bronze gear]

$\mu = 0.028$, using $f = 0.103\, e^{(-0.11 V_s^{0.45})} + 0.012$

$f = 0.026$

$$\Rightarrow e = \frac{\cos 25 - 0.028 \tan 28.62}{\cos 25 + 0.028 \cot 28.62} = 93\ \%$$

7- $b \leq 0.5\,(d_w)_{out}$.

$(d_w)_{out} = d_w + 2a$, a = addendum , From table [15-8]

$\Rightarrow a = (0.286)P$, for $\phi_n = 25°$

$\Rightarrow a = 0.286 \times (0.75) = 0.2145 \Rightarrow (d_w)_{out} = 1.75 + 2(0.2145) = 2.179''$

$b = 0.5(d_w)_{out} = 1.09''$ $\rightarrow$ take $b = 1.0''$ in

$b_g = \frac{2 d_w}{3}$, $P > 0.16''$ $\rightarrow b_g = 1.166 \rightarrow b = 1''.0$

$= 0.93 \times (\ ) = 19.72$ hp.

Note: By increasing cooling capacity → out put power increases.

12 - Friction losses:

$H_f = \frac{F_f V_s}{33000}$, $F_f = \frac{\mu F_{gt}}{\cos\phi_n \cos\lambda - \mu \sin\lambda} = \frac{0.028 \times 1575}{\cos 25 \cos 23.6 - 0.028 \times \sin}$

$= 56.38$ lb

$H_f = \frac{30 \times 626}{33000} = 1.07$

$e = \frac{\text{out put}}{\text{Input}} = \frac{H_{in} - H_f}{H_{in}} \Rightarrow H_{in}(1-e) = H_f = 1.001$

$\Rightarrow H_{in} = \frac{H_f}{1-e}$

$H_{out} = e H_{in} = 14.3$ hp. $H_f = H_o \frac{[1-e]}{e} = 1.076$ hp

$= 35.52$ ft.lb/min.

Find temp. raise

No fan. $C = \frac{n_w}{6494} + 0.13 = 0.315 \frac{ft.lb}{min.F^o.in^2}$.

$A = 43.2\, C^{1.7} = 43.2\,(6.125)^{1.7} = 940.94$. in^2.

$H = CA\,\Delta T = H_f$

$H_f = F_f V_s \rightarrow \Delta T = \frac{F_f V_s}{CA} = \frac{56 \times 626}{0.315 \times 940.94} = 118.27\ F^o$

If $T_a = 70 \rightarrow T_o = 48.27\ F^o$