

Remark

$X_1, \dots, X_n$ iid $N(\mu, \sigma^2)$

$$\frac{n S_n^2}{\sigma^2} \sim \chi^2(n-1)$$

$$E\left(\frac{n S_n^2}{\sigma^2}\right) = n-1$$

$$E(S_n^2) = \left(\frac{n-1}{n}\right) \sigma^2 \neq \sigma^2$$

$$S_{n-1}^2 = \frac{\sum (X_i - \bar{X})^2}{n-1} = \frac{n}{n-1} \frac{\sum (X_i - \bar{X})^2}{n}$$

$$S_{n-1}^2 = \left(\frac{n}{n-1}\right) S_n^2$$

$$E(S_{n-1}^2) = \frac{n}{n-1} \cdot \frac{(n-1)}{n} \cdot \sigma^2$$

$$E(S_{n-1}^2) = \sigma^2$$

* S_n^2 biased estimator for σ^2

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