سمية

Principles of Physics (10th edition) 141

(Discussion)

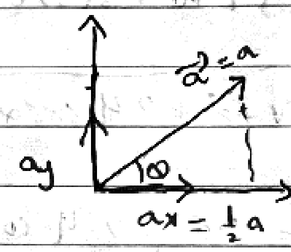
Chapter 3: Vectors

Problems: 1, 7, 15, 32, 35, 44

Problem 1: If the x component of a vector $\vec{a}$, in the xy plane, is half as large as the magnitude of the vector, find the tangent of the angle between the vector and the x axis.

sol: $a_x = \frac{1}{2} a$

صحيح a_x نص قيمة المتجه a



$\cos\theta = \frac{\text{المجاور}}{\text{الوتر}}$

$\cos\theta = \frac{a_x}{a}$

$\cos\theta = \frac{0.5\,a}{a} \Rightarrow \cos\theta = 0.5 \Rightarrow \boxed{\theta = 60}$

لكن السؤال طلب $\tan\theta$ لذلك

$\tan 60 = \sqrt{3} = 1.7$

Problem 7: consider two displacements, one of magnitude 3m and another of magnitude 4m. Show how the displacements vectors may be combined to get a resultant displacement of magnitude (a) 7m, (b) 1m, (C) 5m

Sol: $\vec{d_1} = 3m$, $\vec{d_2} = 4m$ $\theta = ??$

(a) $\vec{d}_{net} = 7m$

لا يعطيني حاصل جمعهم ٧ حتى تكون بأكبر قيمة

$\theta = 0$

$d_{net} = \sqrt{d_1^2 + d_2^2 + 2 d_1 d_2 \cos\theta}$

$7 = \sqrt{9 + 16 + 2(3)(4)\cos\theta}$

$7 = \sqrt{25 + 24\cos\theta}$

تابع

$$\begin{aligned} 49 &= \cancel{25} + 24\cos\theta \\ -25 &\quad \cancel{-25} \end{aligned}$$

$$24 = 24\cos\theta$$

$$\frac{24}{24} = \cos\theta \Rightarrow \cos\theta = 1 \Rightarrow \theta = \cos^{-1}(1) = 0$$

so when d_1, d_2 in same direction

$(\theta = 0) \Rightarrow d_{net} = 7m$

b) $d_{net} = 1$

بما أن حاصل جمع المتجهين 1 أي

كأنهم طُرحوا ← معناه

المتجهين عكس بعض ∴ $\theta = 180$

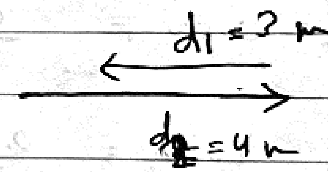
$$1 = \sqrt{9 + 16 + 2(3)(4)\cos\theta}$$

$$1 = \sqrt{25 + 24\cos\theta}$$

ربع الطرفين

$$1 = 25 + 24\cos\theta$$

$$-25 \quad -25$$



$$-24 = 24\cos\theta \Rightarrow \cos\theta = -1 \Rightarrow \boxed{\theta = 180}$$

c) $d_{net} = 5m$

$$5 = \sqrt{9 + 16 + 2(3)(4)\cos\theta}$$

$$5 = \sqrt{25 + 24\cos\theta}$$

ربع الطرفين

$$25 = 25 + 24\cos\theta$$

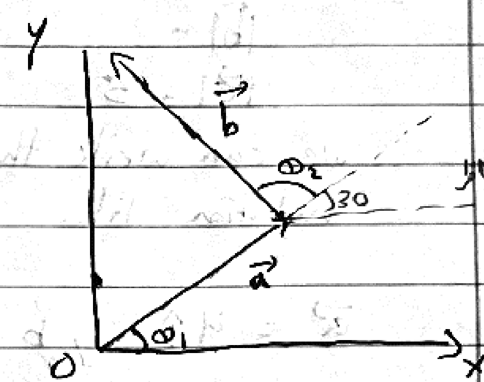
$$-25 \quad -25$$

$$0 = 24\cos\theta \Rightarrow \cos\theta = \text{zero} \Rightarrow \theta = 90$$

اي أن المتجهين متعامدين

Problem 15: The two vectors $\vec{a}$ and $\vec{b}$ in Fig 3.23 have equal magnitudes of 10.0 m and the angles are $\theta_1 = 30°$ and $\theta_2 = 105°$. Find the ⓐ x and ⓑ y componets of their vector sum $\vec{r}$ ⓒ the magnitude of $\vec{r}$ and ⓓ the angle $\vec{r}$ makes with the positive direction of the x axis.

a) x component $\Rightarrow r_x$



$$\vec{r}_x = \vec{a}_x + \vec{b}_x$$
$$= a\cos\theta_1 + b\cos(30+\theta_2)$$
$$= 10\cos 30 + 10\cos(135)$$
$$= 1.59\text{ m}$$

b) y component $\Rightarrow r_y$

$$\vec{r}_y = \vec{a}_y + \vec{b}_y$$
$$= a\sin\theta_1 + b\sin(\theta_2 + 30)$$
$$= 10\sin 30 + 10\sin 135$$
$$= 12.1\text{ m}$$

c) the magnitude of $\vec{r}$ is $|\vec{r}|$ or r

$$r = \sqrt{r_x^2 + r_y^2} = \sqrt{(1.59)^2 + (12.1)^2} = \sqrt{149}$$
$$r = 12.2\text{ m}$$

d) the angle $\vec{r}$ with +ve x

$$\tan\theta = \frac{r_y}{r_x}$$

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{12.1}{1.59}\right)$$

$$\theta = \tan^{-1}(7.6) = 82.5$$

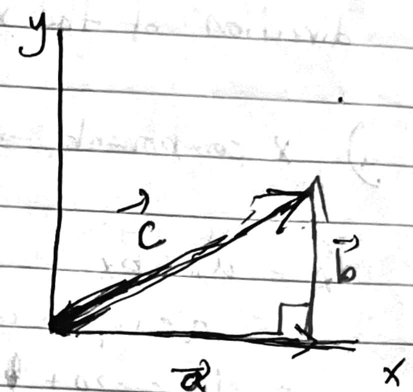
ساره زيا

Problem 32: For the vector in Fig 3-26, with $a=4$, $b=3$ and $c=5$, What are (a) the magnitude (b) the direction of $\vec{a}\times\vec{b}$ (c) the magnitude (d) the direction of $\vec{a}\times\vec{c}$ and (e) the magnitude and (f) the direction of $\vec{b}\times\vec{c}$ (The z axis is not shown)

sol: $|\vec{a}| = 4$

$|\vec{b}| = 3$

$|\vec{c}| = 5$



$\therefore$ we can write them in vector notation like this $\Rightarrow$

$\vec{a} = 4\hat{i}$, $\vec{b} = 3\hat{j}$, $\vec{c} = 4\hat{i} + 3\hat{j}$

a) magnitud of $\vec{a}\times\vec{b}$

$|\vec{a}\times\vec{b}| = |4\hat{i}\times 3\hat{j}| = |12\hat{k}| = 12$

b) direction toward $\hat{k}$ (+z - axis)

اصابعي مع $\hat{i}$
بلف باتجاه $\hat{j}$
ابهامي ليرا $\hat{k}$
او ابهامي مع $\hat{i}$
اصابعي مع $\hat{j}$
كف يدي ليرا $\hat{k}$

c) magnitude of $\vec{a}\times\vec{c}$

$\vec{a}\times\vec{c} = 4\hat{i}\times(4\hat{i}+3\hat{j})$

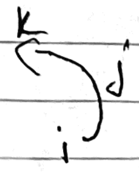
$= 4\hat{i}\times 4\hat{i} + 4\hat{i}\times 3\hat{j}$

$=$ zero $+ 12\hat{k}$

$|\vec{a}\times\vec{c}| = 12$

d) Direction toward $\hat{k}$ (+z axis)

e) magnitude of $\vec{b}\times\vec{c}$



$\vec{b}\times\vec{c} = 3\hat{j}\times(4\hat{i}+3\hat{j})$

$= 3\hat{j}\times 4\hat{i} + 3\hat{j}\times 3\hat{j}$

$= 12(-\hat{k}) +$ zero

$\Rightarrow |\vec{b}\times\vec{c}| = 12$

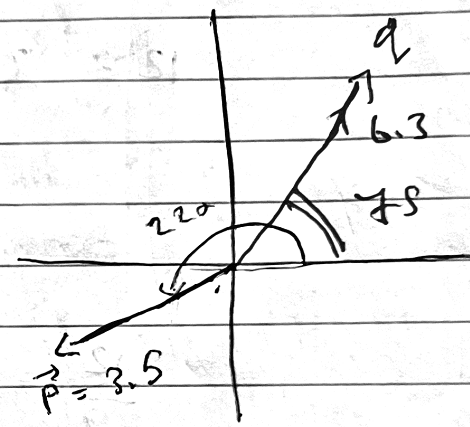
(f) direction toward $(-\hat{k})$

[illegible]

Problem 35: Two vector $\vec{p}$ and $\vec{q}$ lie in the xy plane. Their magnitudes are 3.50 and 6.30 units respectively, and their directions are 220° and 75° respectively, as measured counterclockwise from the positive x-axis. What are the values of a) $\vec{p} \times \vec{q}$ and (b) $\vec{p} \cdot \vec{q}$

sol: ① between $q, b = 220 - 75 = 145°$

a) $\vec{p} \times \vec{q} = pq \sin\theta$
$= 3.5 \times 6.3 \times \sin 145$
$= 12.6$ units $(\hat{k})$



b) $\vec{p} \cdot \vec{q} = pq \cos\theta$
$= 3.5 \times 6.3 \cos 145$
$= -18.1$ units

Problem 44: In the product $\vec{F} = q\vec{v} \times \vec{B}$, take $q = 3$,
$\vec{v} = 2.0\hat{i} + 4.0\hat{j} + 6.0\hat{k}$ and $\vec{F} = 4.0\hat{i} - 20\hat{j} + 12\hat{k}$

What then is $\vec{B}$ in unit-vector notation if $B_x = B_y$

عدل على السؤال $q=2$ مع تسهيل التقاطع مع الارقام

sol: $F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ V_x & V_y & V_z \\ B_x & B_y & B_z \end{vmatrix}$

$4\hat{i} - 20\hat{j} + 12\hat{k} = 2 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 6 \\ B_x & B_y & B_z \end{vmatrix}$

6

ساموكار

$$4\hat{i} - 20\hat{j} + 12\hat{k} = 2(4B_z - 6B_y)\hat{i} - 2(2B_z - 6B_x)\hat{j} + 2(2B_y - 4B_x)\hat{k}$$

$$4 = 2(4B_z - 6B_y) \quad \text{—— } ①$$

$$-20 = -2(2B_z - 6B_x) \quad \text{—— } 2$$

$$12 = 2(2B_y - 4B_x) \quad \text{—— } ③$$

$$B_y = B_x$$

$$\frac{12}{2} = 2B_y - 4B_x$$

$$6 = 2B_x - 4B_x$$

$$6 = -2B_x \Rightarrow B_x = \frac{6}{-2} = -3$$

$$\boxed{B_x = B_y = -3}$$

عوض B في ①

$$4 = 3(4B_z - 6 \times -2)$$

$$4 = 12B_z + 12$$

عوض B_x في معادلة ②

$$-20 = -2(2B_z - 6 \times -3)$$

$$10 = 2B_z + 18 \Rightarrow -8 = 2B_z$$

$$\Rightarrow \boxed{B_z = -4}$$

او في ①

$$4 = 2(4B_z - 6 \times -3)$$

$$2 = 4B_z + 18$$

$$-16 = 4B_z \Rightarrow \boxed{B_z = -4}$$

حل بـ

Probem 44 : $F = q\vec{U}\times\vec{B}$, $q = 3$

$\vec{U} = 2.0\hat{i} + 4.0\hat{j} + 6.0\hat{k}$

$F = 4.0\hat{i} - 20\hat{j} + 12\hat{k}$

$B_x = B_y$

sol.

$$F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ V_x & V_y & V_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$4\hat{i} - 20\hat{j} + 12\hat{k} = 3 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 6 \\ B_x & B_y & B_z \end{vmatrix}$$

$$4\hat{i} - 20\hat{j} + 12\hat{k} = 3(4B_z - 6B_y)\hat{i} - 3(2B_z - 6B_x)\hat{j} + 6(2B_y - 4B_x)\hat{k}$$

$\Rightarrow$

$4 = 3(4B_z - 6B_y)$ —— 1

$-20 = -3(2B_z - 6B_x)$ —— 2

$12 = \cancel{3}(2B_y - 4B_x)$ —— 3

$12 = \cancel{3}(2B_y - 4B_x)$ معادلة 3

$12 = 3(2B_x - 4B_y)$

$\cancel{4} = -2B_y \Rightarrow \boxed{B_y = -2}$ $\boxed{B_x = -2}$

سامي بار

عوض في معادله ①

$$4 = 3(4B_z - 6B_y)$$

$$\frac{4}{3} = 4B_z - 6\times -2$$

$$\frac{4}{3} = 4B_z + 12$$

$$\frac{4}{3} - 12 = 4B_z$$

$$-10.\overline{6} = 4B_z$$

$$\boxed{-2.\overline{6} = B_z}$$

او في معادله ②

$$-20 = -3(2B_z - 6B_x)$$

$$\frac{20}{3} = (2B_z - 6B_x)$$

$$\frac{20}{3} = (2B_z - 6\times -2)$$

$$\frac{20}{3} = 2B_z + 12$$

$$\frac{20}{3} - 12 = 2B_z$$

$$-5.\overline{3} = 2B_z$$

$$-2.\overline{6} = B_z$$

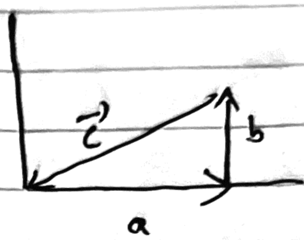
(9)

Problem 32

$a = 4\hat{i}$

$b = 3\hat{j}$

$\vec{c} = -4\hat{i} - 3\hat{j}$



a) $|\vec{a} \times \vec{b}| = |4\hat{i} \times 3\hat{j}| = 12$

b) $\hat{k}$

c) $|\vec{a} \times \vec{c}| = 4\hat{i} \times (-4\hat{i} - 3\hat{j})$

$= 4\hat{i} \times -4\hat{i} - 4\hat{i} \times +3\hat{j}$

$= \text{zero} - (4\hat{i} \times 3\hat{j})$

$= -(12\hat{k})$

$= -12\hat{k}$

$|\vec{a} \times \vec{c}| = 12$

d) $-\hat{k}$

e) $|\vec{b} \times \vec{c}| = 3\hat{j} \times (-4\hat{i} - 3\hat{j})$

$= 3\hat{j} \times -4\hat{i} - 3\hat{j} \times 3\hat{j}$

$= -(3\hat{j} \times 4\hat{i}) - \text{zero}$

$= -(-12\hat{k})$

$= +12\hat{k}$

$|b \times c| = 12$

f) $\hat{k}$