8.5 Young's Two-Slit Setup and Spatial ⑦ coherence.
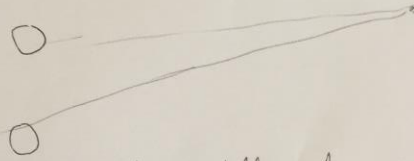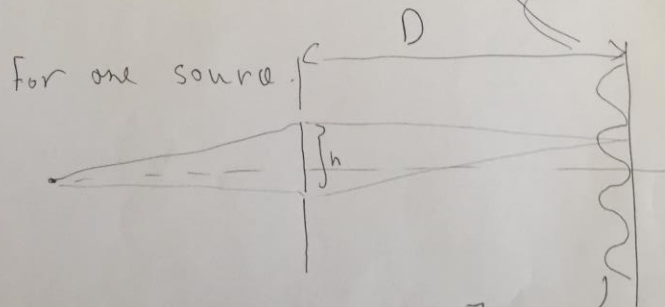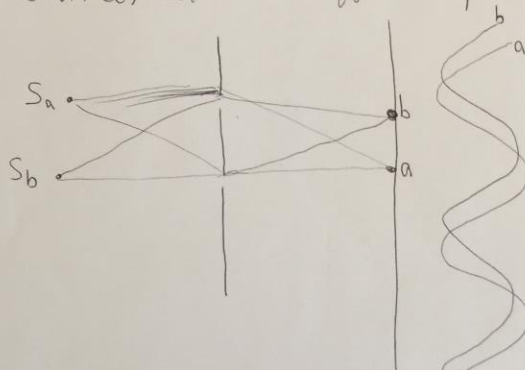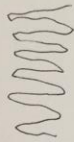

Two sources at two different points.

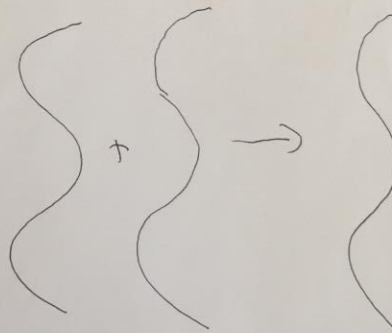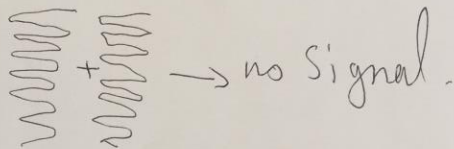for one source.

$$I_{tot}(h) = 2I_0 \left[1 + \cos(kyh/D) \right]$$

IF $h \to$ large, the fringe pattern
becomes dense.

This results in Cancellation.

+ → no signal.

↑ →

There is
interference

9.1 The Eikonal Equation

Assume light with one frequency isotropic medium; real refractive index.

From wave equation:

$$\nabla^2 \vec{E}(\vec{r},t) + [n(\vec{r})]^2 \frac{\omega^2}{c^2} \vec{E}(\vec{r},t) = 0$$

<u>Note</u>: $n(\vec{r})$ depends on position

By analogy let the trial solution be

$$\vec{E}(\vec{r},t) = \vec{E_0}(\vec{r}) e^{i[k_{vac} R(\vec{r}) - \omega t]}$$

$$k_{vac} = \frac{\omega}{c} = \frac{2\pi}{\lambda_{vac}}$$

Subbing the solution we get

$$\frac{1}{k_{vac}^2} \nabla^2 \left[\vec{E_0}(\vec{r}) e^{i k_{vac} R(\vec{r})} \right] + [n(\vec{r})]^2 \vec{E_0}(\vec{r}) e^{i k_{vac} R(\vec{r})} = 0$$

In very short λ limit, we get

$$\nabla R \cdot \nabla R = [n(\vec{r})]^2$$

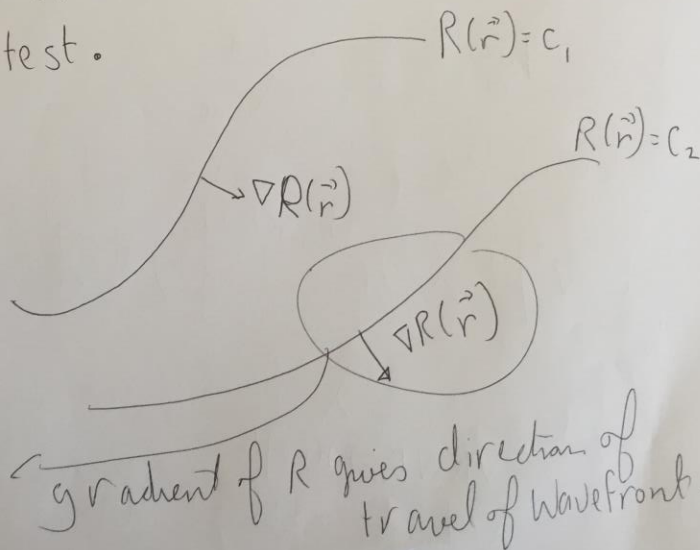(9.7) $$\nabla R(\vec{r}) = n(\vec{r}) \hat{s}(\vec{r})$$

$\hat{S}$ = unit vector in direction of $\nabla R(\vec{r})$

= direction normal to wavefront surfaces.

$\nabla R(\vec{r}) = n(\vec{r}) \, \hat{s}(\vec{r})$ $\left[\text{Eikonal equation}\right]$

What is $R(\vec{r})$? Real scalar function

$R(\vec{r})$ = constant = Wavefront.

This wavefront travels in direction for which $R(\vec{r})$ changes the fastest.

$R(\vec{r}) = C_1$

$R(\vec{r}) = C_2$

$\nabla R(\vec{r})$

$\nabla R(\vec{r})$

gradient of R gives direction of travel of wavefront.

Example 9.1

let $n(y) = n_0 \sqrt{1 + y^2/h^2}$

Verify that $R(x,y) = n_0 (x \pm \dot{y}^2/2h)$
is a solution to the eikonal
equation.

$$\nabla R = n_0 \left(\hat{x} \pm \frac{y}{h} \hat{y} \right)$$
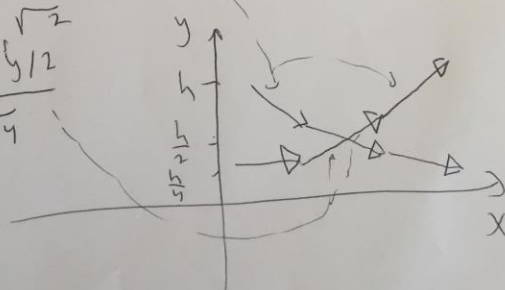
$$\nabla R \cdot \nabla R = n_0^2 \left(1 + \frac{y^2}{h^2} \right) = [n(y)]^2$$

The direction of light propagation is

$$\hat{S}(y) = \frac{\nabla R}{|\nabla R|} = \frac{\hat{x} \pm \hat{y} \frac{y}{h}}{\sqrt{1 + y^2/h^2}}$$

$$\hat{S}(h) = \frac{\hat{x} \pm \hat{y}}{\sqrt{2}}$$

$$\hat{S}\left(\frac{h}{2}\right) = \frac{\hat{x} \pm \hat{y}/2}{\sqrt{5/4}}$$

9.2 Fermat's Principle

Used to find path of rays.

A ⟶ B

follows a path that takes least time.

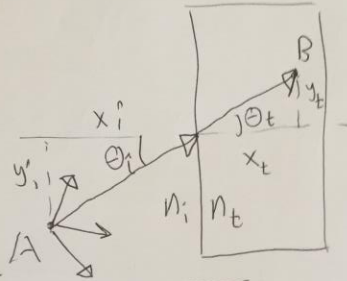$$OPL \Big|_A^B = \min \left\{ \int_A^B n \, d\ell \right\}$$

$$\Delta t \Big|_A^B = \int_A^B \frac{d\ell}{U(\vec{r})} = \int_A^B \frac{d\ell}{c/n(\vec{r})} = \frac{OPL \Big|_A^B}{C}$$

To find correct path of light, we minimize the OPL.

Example 9.2

Derive Snell's law using Fermat's Principle.

$$OPL = n_i \sqrt{x_i^2 + y_i^2} + n_t \sqrt{x_t^2 + y_t^2}$$

This should be minimized.

x_i, x_t = fixed.

$y_{tot} = y_i + y_t$ = constant.

$$OPL = n_i \sqrt{x_i^2 + y_i^2} + n_t \sqrt{x_t^2 + (y_{tot} - y_i)^2}$$

only variable is y_i.

$$\frac{d(OPL)}{dy_i} = n_i \frac{y_i}{\sqrt{x_i^2 + y_i^2}} + n_t \frac{[-(y_{tot} - y_i)]}{\sqrt{x_t^2 + (y_{tot} - y_i)^2}} = 0$$
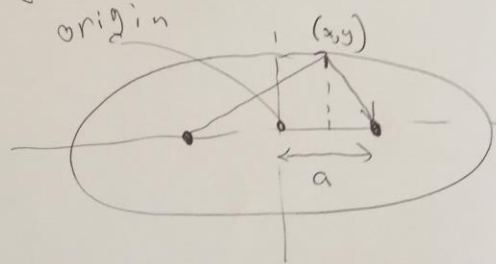
$$\sin \theta_i = \frac{y_i}{\sqrt{x_i^2 + y_i^2}} \quad ; \quad \sin \theta_t = \frac{y_t}{\sqrt{x_t^2 + y_t^2}}$$

$$n_i \sin \theta_i - n_t \sin \theta_t = 0$$

$$\boxed{n_i \sin \theta_i = n_t \sin \theta_t}$$

Example 9.3   Derive equation of curvature
for a reflective surface that causes
all rays leaving one point to image
to another. (In 2D).

origin

(x,y)

a

$$\sqrt{(x+a)^2 + y^2} + \sqrt{(x-a)^2 + y^2} = b$$
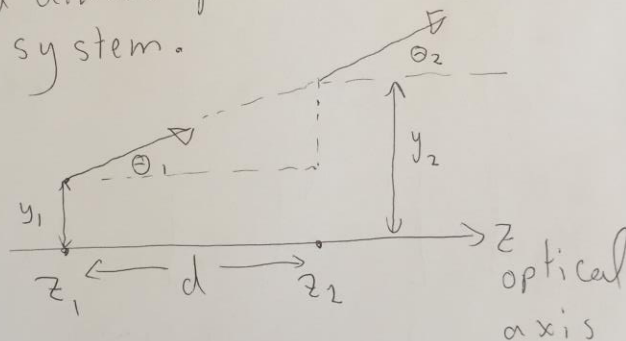
[after some algebra] [

we get

$$\frac{x^2}{\left(\frac{b^2}{4}\right)} + \frac{y^2}{\left(\frac{b^2}{4} - a^2\right)} = 1 \quad ; \text{ellipse}.$$

9.3 Paraxial Rays and ABCD matrices $\boxed{8}$

- **Ray Optics:** Describe light as rays and not waves.

  Good approximation if $\lambda \ll$ features.

- **Paraxial Approximation:** rays near and almost parallel to optical axis of system.

$\sin\Theta \cong \Theta$

$\tan\Theta \cong \Theta$

let n be uniform everywhere.

$$\Theta_2 = \Theta_1$$
$$y_2 = y_1 + d\tan\Theta_1 = y_1 + \Theta_1 d$$

$$\therefore \begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

↑

ABCD matrix for propagation through a distance d.

Example 9.4   let $d = a + b$

show that applying ABCD matrix for $\underline{\underline{d}}$, or for $\underline{\underline{a}}$ then b, gives same result.

$$\begin{bmatrix} y_{mid} \\ \theta_{mid} \end{bmatrix} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_{mid} \\ \theta_{mid} \end{bmatrix}$$

$$\therefore \begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & a+b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix} \quad \checkmark$$