Example

matrix	A1	A2	A3	A4	A5	A6
dimension	4x2	2x3	3x1	1x2	2x2	2x3

Table m

M	A1	A2	A3	A4	A5	A6
A1	0	24	14	22	26	36
A2		0	6	10	14	22
A3			0	6	10	19
A4				0	4	10
A5					0	12
A6						0

	2	3	4	5	6
1	1	1	3	3	3
2		2	3	3	3
3			3	3	3
4				4	5
5					5

Computeng the Optimal Costs :-

l=2

m[1,2], m[2,3], m[3,4], m[4,5], m[5,6]

l=3

m[1,3], m[2,4], m[3,5], m[4,6]

Compute each one and store the min value in the table m

l=4

m[1,4], m[2,5], m[3,6]

l=5

m[1,5], m[2,6]

l=6

m[1,6]

$$m[1,2] = 4\times2\times3 = 24$$

$$m[2,3] = 2\times3\times1 = 6$$

$$m[3,4] = 3\times1\times2 = 6$$

$$m[4,5] = 1\times2\times2 = 4$$

$$m[5,6] = 2\times2\times3 = 12$$

$$m[1,3] = \min\begin{cases} \overset{0}{m[1,1]} + \overset{\overset{*}{6}}{m[2,3]} + \overset{*}{P_0 P_1} \overset{*}{P_3} = 0 + 6 + 4\times2\times1 = \boxed{14} \quad k=1 \\ m[1,2] + m[3,3] + P_0 P_2 P_3 = 24 + 0 + 4\underline{\times3}\times1 = 36 \end{cases}$$

$$m[2,4] = \min\begin{cases} m[2,2] + m[3,4] + P_1 \overset{\times}{P_2} \overset{\times}{P_4} = 0 + 6 + \overset{2\times3\times2}{12} = 18 \\ m[2,3] + m[4,4] + P_1 P_3 P_4 = 6 + 0 + \underset{2\times1\times2}{4} = \boxed{10} \quad k=3 \end{cases}$$

$$m[3,5] = \min\begin{cases} m[3,3] + m[4,5] + \overset{3\times1\times2}{P_2 P_3 P_5} = 0 + 4 + 6 = \boxed{10} \quad k=3 \\ m[3,4] + m[5,5] + P_2 P_4 P_5 = 6 + 0 + 12 = 18 \end{cases}$$

$$m[4,6] = \min\begin{cases} m[4,4] + m[5,6] + P_3 P_4 P_6 = 0 + 12 + 6 = 18 \\ m[4,5] + m[6,6] + P_3 P_5 P_6 = 4 + 0 + 6 = \boxed{10} \quad k=5 \end{cases}$$

$$m[1,4] = \min\begin{cases} \overset{*}{\min[1,1]} + \overset{*}{m[2,4]} + \overset{*}{P_0}\times P_1\times P_4 = 0 + 10 + \overset{4\times2\times2}{16} = 26 \\ \min[1,2] + m[3,4] + P_0\times P_2\times P_4 = 24 + 6 + 24 = 54 \\ \min[1,3] + m[4,4] + P_0\times P_3\times P_4 = 14 + 0 + 8 = \boxed{22} \quad k=3 \end{cases}$$

$$m[2,5] = \min\begin{cases} m[2,2] + m[3,5] + P_1 P_2 P_5 = 0 + 10 + \overset{2\times3\times2}{12} = 22 \\ m[2,3] + m[4,5] + P_1 P_3 P_5 = 6 + 4 + 4 = \boxed{14} \quad k=3 \\ m[2,4] + m[5,5] + P_1 P_4 P_5 = 10 + 0 + 8 = 18 \end{cases}$$

$$m[3,6] = \min\begin{cases} m[3,3] + m[4,6] + P_2 P_3 P_6 = 0 + 10 + \overset{3\times1\times3}{9} = \boxed{19} \quad k=3 \\ m[3,4] + m[5,6] + P_2 P_4 P_6 = 6 + 12 + \overset{3\times2\times3}{18} = 36 \\ m[3,5] + m[6,6] + P_2 P_5 P_6 = 10 + 0 + \overset{3\times2\times3}{18} = 28 \end{cases}$$

$$m[1,5] = \min \begin{cases} m[1,1] + m[2,5] + P_0 P_1 P_5 = 0+14+16 = 30 \\ m[1,2] + m[3,5] + P_0 P_2 P_5 = 24+10+24 = 58 \\ m[1,3] + m[4,5] + P_0 P_3 P_5 = 14+4+8 = \boxed{26} \; k=3 \\ m[1,4] + m[5,5] + P_0 P_4 P_5 = 22+0+16 = 38 \end{cases}$$

$$m[2,6] = \min \begin{cases} m[2,2] + m[3,6] + P_1 P_2 P_6 = 0+19+18 = 37 \\ m[2,3] + m[4,6] + P_1 P_3 P_6 = 6+10+6 = \boxed{22} \; k=3 \\ m[2,4] + m[5,6] + P_1 P_4 P_6 = 10+12+12 = 34 \\ m[2,5] + m[6,6] + P_1 P_5 P_6 = 14+0+12 = 26 \end{cases}$$

* * *

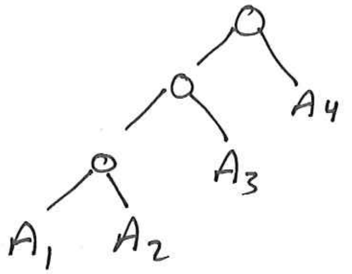
$$m[1,6] = \min \begin{cases} m[1,1] + m[2,6] + P_0 P_1 P_6 = 0+22+24 = 46 \\ m[1,2] + m[3,6] + P_0 P_2 P_6 = 24+19+36 = 79 \\ m[1,3] + m[4,6] + P_0 P_3 P_6 = 14+10+12 = \boxed{36} \; k=3 \\ m[1,4] + m[5,6] + P_0 P_4 P_6 = 22+12+24 = 58 \\ m[1,5] + m[6,6] + P_0 P_5 P_6 = 26+0+24 = 50 \end{cases}$$

Example:- Matrix Chain Multiplication Problem

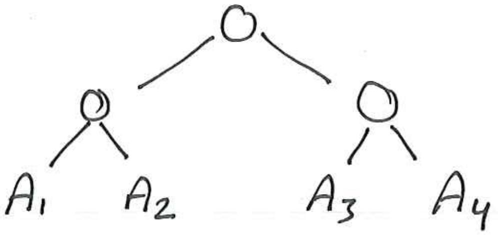
A_1	A_2	A_3	A_4
5x4	4x6	6x2	2x7

How to multiply?

$((A_1 \cdot A_2) \cdot A_3) \cdot A_4)$



$(A_1 \cdot A_2) \cdot (A_3 \cdot A_4)$



How to find the minimum number of multiplications to multiply the 4 matrecies?

* Using Dynamic Programing

$L = 1 \Rightarrow$ All zeros

✓ A_1, A_2, A_3, A_4

$L = 2$

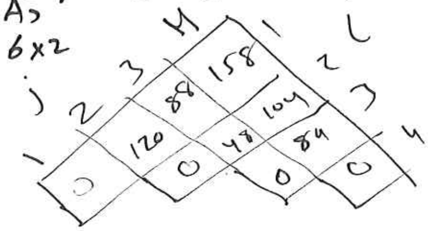
$m[1,2]$, $m[2,3]$, $m[3,4]$

$A_1 \cdot A_2$ A_2 4x6 A_3 6x2

4

$5 \times \boxed{4 \quad 4} \times 6$

$\uparrow \quad \uparrow \quad \uparrow$



m	1	2	3	4
1	0	120	88	158
2		0	48	104
3			0	84
4				0

S	1	2	3	4
1		1	1	3
2		~~2~~	2	3
3				3
4				

$L : 3$

$m[1,3]$,

$(A_1 (A_2 \; A_3))$

5x4

$m[1,1] + m[2,3] + 5 \times 4 \times 2$

A_1	A_2	A_3
5×4	4×6	6×2

$= 48 + 40$

$= 88$

5x4   4x2

$5 \times 4 \times 2$

$((A_1 \; A_2) \cdot A_3)$

5x4   4x6   6x2

$m[1,2] + m[3,3] + 5 \times 6 \times 2$

$120 + 0 + 120$

(80)

✓ 88

$m[2,4] = 104$

$104 \rightarrow k=3$

$A_2\ (A_3\ A_4)$

$4\times6 \quad 6\times2 \quad 2\times7$

$m[2,2]+m[3,4]+4\times6\times7$

$0 + 84 + 168 = 252$

$(A_2\ A_3)\ A_4$

$m[2,3] + m[4,4] + 4\times2\times7$

$48 + 0 + 56$

104

$l = 3$

$A_1 \quad A_2 \quad A_3 \quad A_4$

$5\times4 \quad 4\times6 \quad 6\times2 \quad 2\times7$

$m[1,4] = \min \left\{ m[1,1] + m[2,4] + 5\times4\times7,\ m[1,2] + m[3,4] + 5\times6\times7 \right.$

$0 + 104 + 140 \qquad 120 + 84 + 210$

$m[1,3] + m[4,4] + 5\times2\times7$

$88 + 0 + 70 = 158$

$k=3$

$s \Rightarrow 3$

Formula

$$m[i,j] = \min \left\{ m[i,k] + m[k+1,j] + P_{i-1} * P_k * P_j \right.$$

$A_1 \quad A_2 \quad A_3 \quad A_4$

$5\times4 \quad 4\times6 \quad 6\times2 \quad 2\times7$

$P_0 \quad \cancel{P_1} \quad P_1 \quad \cancel{P} \quad P_2 \quad P_3 \quad P_4$

time complexity

because of zeros

$$\frac{n(n-1)}{2} = n^2 \times n \Rightarrow O(n^3)$$

we calculate only half of the table

for each value in the table, we find all possible values, n times

0			
	0		
		0	
			0

m

	0	1	2	3	4
0	0	0	0	0	0
1	0	0			
2	0		0		
3	0			0	
4	0				0

s

	0	1	2	3	4
0	0	0	0	0	0
1	0	0			
2	0		0		
3	0			0	
4	0				0

i	j
1	2
2	3
3	4

differency

d = 1

i	j
1	3
2	4

d = 2

i	j
1	4

d = 3

calculate diagonally

so the most outerloop becomes: for(d=1 ; d< n-1 ; d++)

n=5

how to find j??

d + i ✓

d< n-1? why n-1 | the first loo time then the row will reach 3 then 2 then 1. 3 is (4-1) dimensions.

```
{
 P[5] = {5, 4, 6, 2, 7}
 m[5][5] = {0};
 s[5][5] = {0};
 int  j, min, q;

 for (int  d = 1; d < n-1 ; d++)
     for ( i = 1; i < n-d ; i++)
     {
         ~~j= [illegible];~~
         j = i+d
          min = ∞ ;
         for (int  k = 1; k <= j-1 ; k++)
         {
             q = m[i][k] + m[k+1][j] + P[i-1] * P[k] * P[j];
             if (q < min)
             {
                 min = q;
                 S[i][j] = k;
             }
         }
             m[i][j] = min;
       }
     }
}
```