

Problem

Let X and Y be sets, let A and B be any subsets of X , and let C and D be any subsets of Y . Determine which of the properties are true for all functions F from X to Y and which are false for at least one function F from X to Y . Justify your answers.

Exercise

$$F(A) \cap F(B) \subseteq F(A \cap B)$$

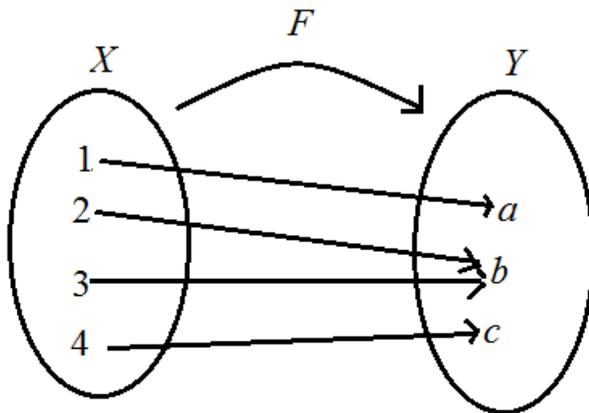
Step-by-step solution

Step 1 of 3

Let X and Y be any sets and A and B be subsets of X and C and D be subsets of Y . The objective is to determine the statement, $F(A) \cap F(B) \subseteq F(A \cap B)$ is true or false.

Consider the sets $X = \{1, 2, 3, 4\}$ and $Y = \{a, b, c\}$.

Define the function $f: X \rightarrow Y$ by the arrow diagram.



Step 2 of 3

Consider the subsets $A = \{1, 2\}$ and $B = \{1, 3\}$.

The function defined on sets $A = \{1, 2\}$ and $B = \{1, 3\}$ as below

$$\begin{aligned} F(A) &= F(\{1, 2\}) \\ &= \{a, b\} \end{aligned}$$

$$\begin{aligned} F(B) &= F(\{1, 3\}) \\ &= \{a, b\} \end{aligned}$$

Step 3 of 3

By the definition of intersection of sets,

$$\begin{aligned} F(A) \cap F(B) &= \{a, b\} \cap \{a, b\} \\ &= \{a, b\} \end{aligned}$$

By the definition of intersection of sets, $A \cap B = \{1\}$ and

$$\begin{aligned} F(A \cap B) &= F\{1\} \\ &= \{a\} \end{aligned}$$

By the definition of subsets, $F(A) \cap F(B) \not\subseteq F(A \cap B)$ because $b \notin \{a\} = F(A \cap B)$.

Hence, by the below counter example the statement, $F(A) \cap F(B) \subseteq F(A \cap B)$, is false.