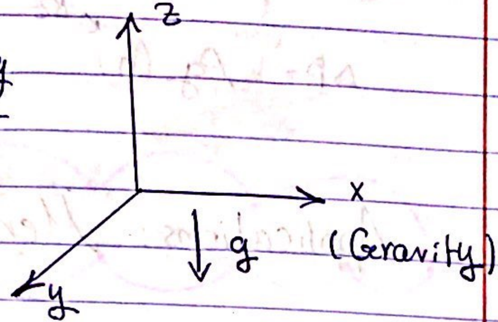
Chapter 2 :
Hydrostatic fluid

2.1 Pressure and Pressure Gradient

- In Hydrostatic fluid as long as you're moving in x, y direction, P_z won't Change : (P is changing vertically)

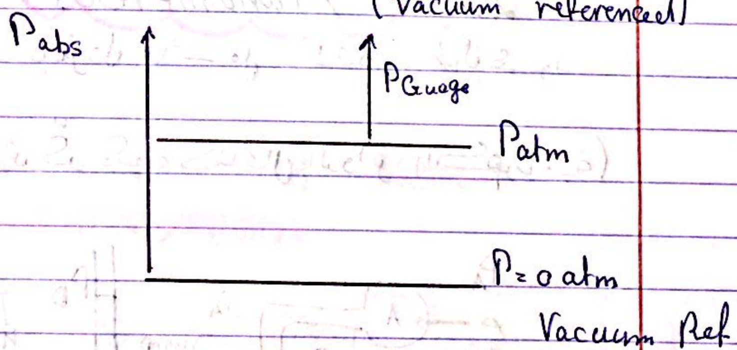


$$P_z = P_n + \frac{1}{2} \rho g \, \Delta z$$

الضغط متعـود
السـائل

2.2 • P_n: is pressure Change in the horizontal Direction and It equals zero here.

2.3 pressure →
- **Guage pressure** = $P_{abs} - P_{atm}$ (atm referenced)
- **absolute pressure** = $P_{guage} + P_{atm}$ (Vacuum referenced)



As we Knew before : $\frac{dp}{dz} = -\gamma$

- تغير الضغط بالنسبة لـ z

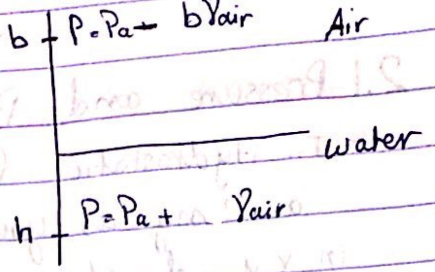
γ → specific weight — تغير وزن عـود السـائل

pressure decreases when we move upward

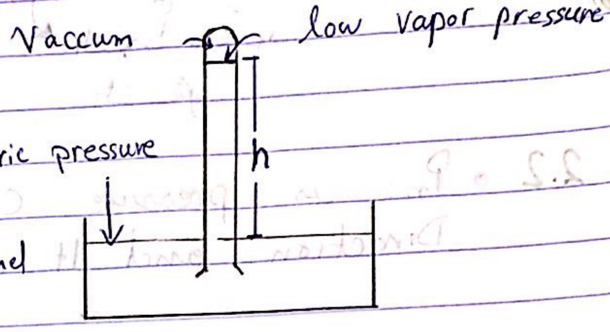
for liquids & from that we conclude that (By solving the differential equation)

$$\Delta z = \frac{\Delta p}{\gamma}$$

$$\Delta p = \rho g \, h^{\,\swarrow \Delta z}$$



Applications: Mercury Barometer



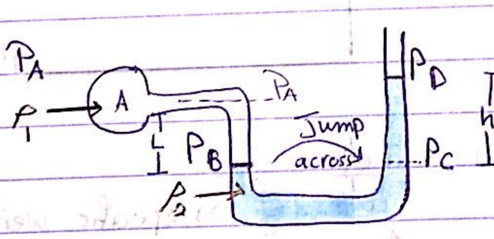
Atmospheric pressure pushes mercury and so mercury rises to height h

2.4 Manometers

تذكر دائماً : الضغط يزداد بالنزول للأسفل لأن طول عمود السائل يزداد.

نعامل الضغط كمتجهات (تذكّر كيف كنا نحل الدوائر الكهربائية)

مثال

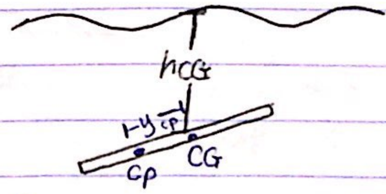


$$P_A + L\rho_1 - h\rho_2 = P_D = P_a$$

Jump across : • معناها لما النقطتين على نفس المستوى، الضغط متساوي عندهم

2.5: hydrostatic forces on plane surfaces

Force on Body From water and atm is



$$F = P_{CG}\, A$$

where: A is the Area of Body

$$P_{CG} = P_{wat} + P_{atm} = \gamma h_{CG} + P_a$$

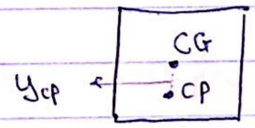
$$y_{cp} = \frac{-\gamma \sin\theta \, I_{xx}}{P_{CG}\, A}$$

$$x_{cp} = -\frac{\gamma \sin\theta \, I_{xy}}{P_{CG}\, A}$$

where: • I_{xx} is moment around X-axis

• θ: Between surface and Body.

⇒ If $I_{xy} = 0$ (Due to symmetry) Then $x_{cp} = 0$ and cp lies under CG Directly



Now: a special case: if $P_{atm} = 0$

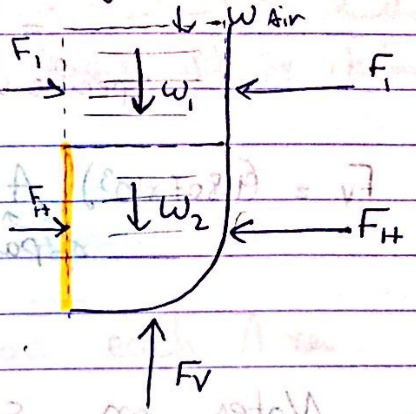
Then $P_{CG} = P_{Guage}$ only

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{CG}\, A}$$

$$x_{cp} = \frac{-I_{xy} \sin\theta}{h_{CG}\, A}$$

2.6 Hydrostatic forces on curved surfaces

for curved surfaces we always use projection



- The wall will exert ~~an~~ force on the fluid which has two components horizontal and vertical

- There are forces opposing These forces

F_1 exerted by the left side of the fluid column opposes F_H.

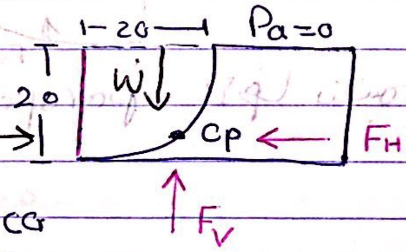
W_1, W_2, W_{Air} are weights opposing F_V

The line in orange is called the projection of the curved path.

Now Remember : F_H, F_V passes through Center of pressure

Example:

The dam below is a quarter circle 50m wide into the paper Determine F_H, F_V.



$\Sigma F_x = 0$

$F_1 = F_H = (\gamma_w)(A_{of\ projection})(h)$

specific weight of water

Area of projection

height of cg on projection

so $F_H = 9.807 \times 10^3 \times 20 \times 50 \times \frac{20}{2} = 98\ MN$

To Find F_V:

$$F_V = W = mg = \rho V g$$
$$= \gamma V$$

γ: specific weight; V: volume of fluid

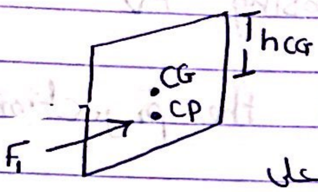
$$\text{so } F_V = (9.807\times10^3)(A)(50) = 9.807\times10^3 \times \left[\frac{1}{4}(\pi)(20)^2\right]\times 50$$

A: of parabola

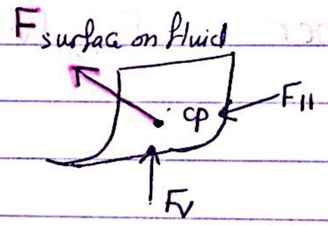
$$= 153.9 \ MN$$

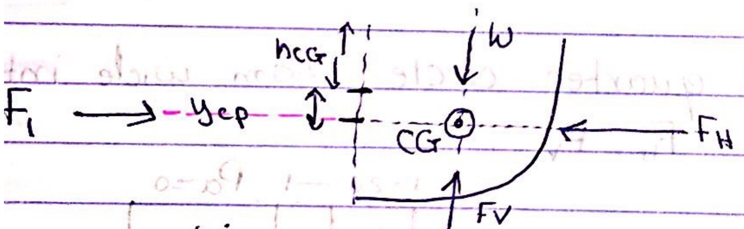
- Notes on Solution:

ليش حطينا hCG بس المسافة العمودية ؟
لأنه أنا باحسب القوة اللي على الـ Projection اللي تعاكس
FH وهاي القوة بتأثر عمودي على السطح
بهذا الشكل



ولنفس الطريقة FH بتمر في CP بس على
الـ curved surface





2D فلو بدي ارسمها

أما FV فتمر بـ CG للـ parapola لأنها بنفس
الخط اللي بتقع عليه W

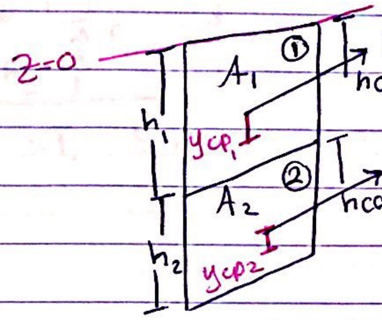
2.7 Hydrostatic forces in layered fluids

• بما أنه عندي طبقات مختلفة من موائع ب ارتفاعات وكثافات
مختلفة فما نستطيع استخدام 2.5
• نتعامل مع كل layer على حدا ونحل كل layer لوحدها

How to find F resultant ?

1 → you need to find P on each Area

First layer :- $P_1 = \gamma h_{ca_1}$



Second layer : $P_2 = \gamma h_1 + \gamma h_{ca_2}$

2 – $F = P A$

$F_1 = P_1 A_2$ $\qquad$ $F_2 = P_2 A_2$

$F = \Sigma F = F_1 + F_2$

3 → you need cp for each area

$y_{cp_1} = \frac{\gamma \sin\theta \, I_{xx_1}}{F_1}$

$y_{cp_2} = \frac{\gamma \sin\theta \, I_{xx_2}}{F_2}$

4 – Specify a reference $\boxed{z=0}$

5 – find z_{cp} for each Area

$z_{cp_1} = h_{ca_1} + y_{cp_1}$

$z_{cp_2} = h_1 + h_{ca_2} + y_{cp_2}$

6- We find z_{cp} for the resultant force using Equation:

$$\sum F_i \, z_{cp_i} = F_R \, z_{cp_R}$$

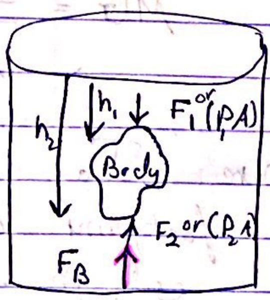
$$F_1 z_{cp_1} + F_2 z_{cp_2} = (F_1 + F_2) \, z_{cp_R}$$ ← Find this

2.8 Bouyancy and stability

Bouyancy Force :-

$F_B = \boxed{\gamma \, v_{Body}}$ ← weight of fluid displaced by the object and F_B passes through center of Bouancy

F_B is the difference Between F_1 and F_2



- Where F_1, F_2 are weights difference of Body due to water weight and F_B is always upward fighting against gravity

$P_2 > P_1$ since $h_2 > h_1$

Example 2.11

Weight in air = 400N

" " water = 240N

نلاحظ انخفاض وزن الجسم في الماء

400 − 240 = 160

هنا الفرق هو وزن الماء المزاح و هو

وهذا الوزن يساوي F_B مقداراً وتعاكسه اتجاهاً

$$F_B = W = \gamma V$$

$$160 = 9.8 \times 10^3 \, V$$

$$V = 0.016 \, m^3$$

So: $\gamma_{Block} = \rho g = \frac{m}{V} g$ ($\frac{400}{g}$) $= \frac{400}{0.016} = 24.5 \, KN/m^3$

Stability related to waterline Area:-

When you have a floating body tilting with Theta (θ) you can check the stability of this Body using equation

$$\overline{MG} = \frac{I_o}{V_{sub}} - \overline{GB}$$

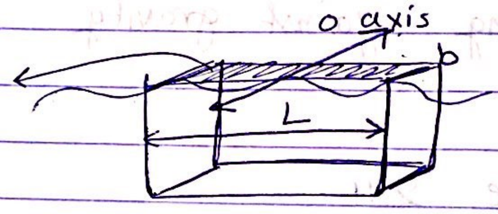
: MG: metacentric height

Where, I_o : Area moment of inertia of the water line footprint around O axis

يعني الـ moment للمساحة اللي بتتحرك على سطح المي

مثلاً

$$I_{o\,here} = \frac{bL^3}{12}$$



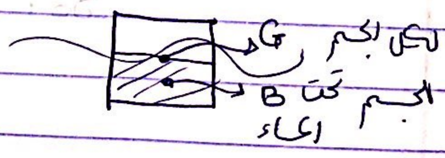
$V_{submerged}$ = مساحة الجسم تحت الماء

$\overline{GB}$: Distance Between Center of Gravity G and center of bouyancy B (B is center of Area under water)

Now: If $\overline{MG} > 0 \longrightarrow$ Stable Body

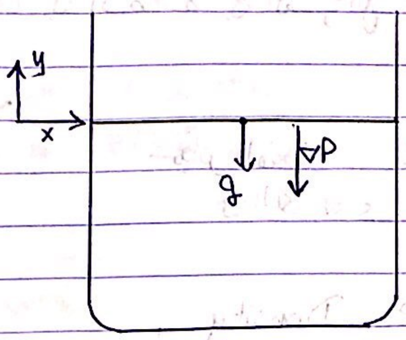
$\overline{MG} < 0 \longrightarrow$ unstable Body

Note: B is found the same way G is found But for Area (Body) under Water.



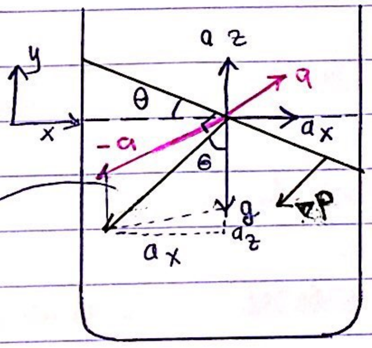
2.9: Uniform linear acceleration

Fluid at rest:

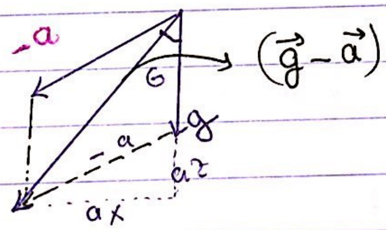


• لما يكون المائع في حالة الراحة تكون عليه تسارع الجاذبية فقط
وبالتالي تكون الزيادة في الضغط لأسفل اتجاهها ($\nabla P \propto g$)

Fluids moving a rigid bodies with an acceleration:-



Parallegram اذا رأينا هذا ال
عن قرب



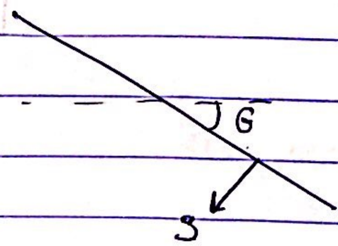
$$\tan\theta = \frac{a_x}{a_z + g}$$

لما يكون المائع تسارع كجسم يكون في تسارع اضافي وهو a هذا
التسارع هو محصلة لتسارعين: افقي a_x وعمودي a_z (مش دائماً موجود)
هذا التسارع اضافة الى تسارع الجاذبية يؤثران على زيادة الضغط
و ∇P تصبح عمودية على السطح المائل وباتجاه ال $\vec{g}-\vec{a}$: $\nabla P \propto (\vec{g}-\vec{a})$
يعني تصير $\nabla P = \rho(\vec{g}-\vec{a})$

$$\frac{dP}{ds} = \rho G$$

هذه المعادلة تعني ان الزيادة
في الضغط في الاتجاه المائل $(g-a)$
تكون أكبر من الزيادة في الضغط في السوائل الساكنة

$\frac{dP}{ds}$ تغير الضغط
في الاتجاه s



ρ: Density

$$\Rightarrow G = \text{محصلة التسارعات} = \sqrt{\left(a_x^2 + (g+a_z)^2\right)}$$

$$P = \rho G \Delta s$$