Ch4

Saturday, March 27, 2021 9:28 AM

Q1 ⓕ $y = f(x) = \frac{x}{x^2-1}$

$\frac{x^{①}}{x^{②}} \Rightarrow \exists$ H. Asy.

$\Rightarrow \nexists$ O. Asy.

- $D(f) \Rightarrow x^2-1=0 \Rightarrow x=\pm 1 \Rightarrow D(f)=\mathbb{R}\setminus\{\pm 1\}$
- Asy: • $\nexists$ O. Asy.
  - H. As: ✓ $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x}{x^2-1} = 0 \Rightarrow y=0$ H. Asy.

    ✓ $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x}{x^2-1} = 0$
  - V. Asy $\Rightarrow$ check zeros for denominator ① check $x=-1$ ② check $x=1$

① $x=-1 \Rightarrow \lim_{x\to-1^+} f(x) = \lim_{x\to-1^+} \frac{x}{x^2-1} = \frac{-1}{\text{small} -} = \boxed{+\infty}$

$\Rightarrow x=-1$ is V. Asy.

-0.9999

$-1.0001 \quad -1$

$\lim_{x\to-1^-} f(x) = \lim_{x\to-1^-} \frac{x}{x^2-1} = \frac{-1}{\text{small} +} = \boxed{-\infty}$

$0.999 \quad 1.0001$

1

② $x=1 \Rightarrow \lim_{x\to1^+} f(x) = \lim_{x\to1^+} \frac{x}{x^2-1} = \frac{1}{\text{small} +} = \boxed{+\infty}$

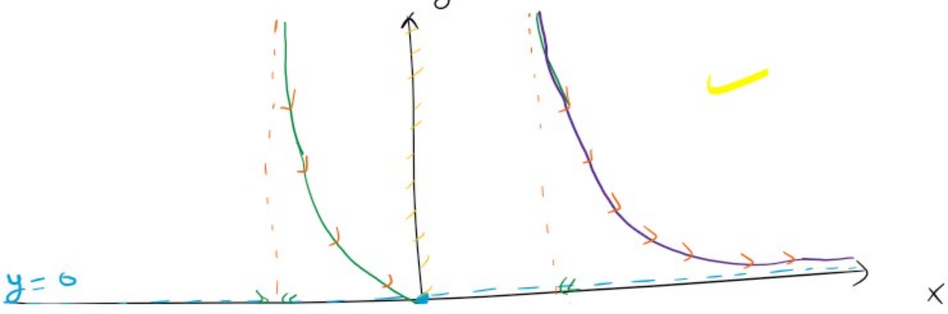
$\Rightarrow x=1$ is V. Asy

$\lim_{x\to1^-} f(x) = \lim_{x\to1^-} \frac{x}{x^2-1} = \frac{1}{\text{small} -} = \boxed{-\infty}$

$f(x) = \frac{x}{x^2-1}$

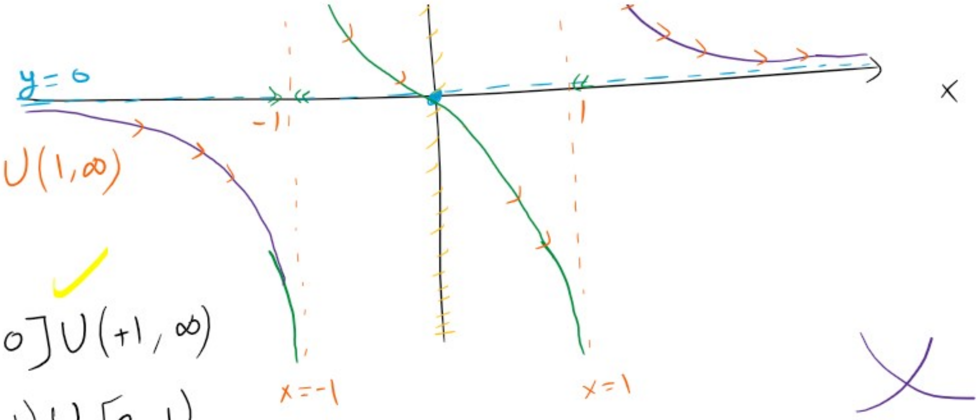
Key point $(0,0)$

f is not increasing



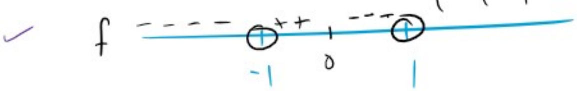
f is not increasing

f ↓ on $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$



f concave up on $(-1,0]\cup(+1,\infty)$ ✓

f concave down on $(-\infty,-1)\cup[0,1)$



Range = $\mathbb{R}$ = $(-\infty,\infty)$

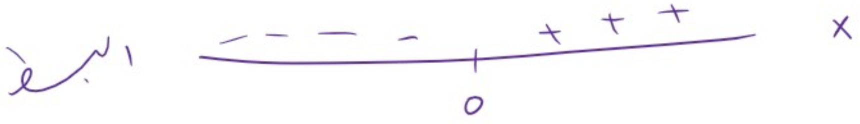
$x=a$ is V. Asy. if $\lim_{x\to a^+} f(x) = \infty$ or $-\infty$ ✓

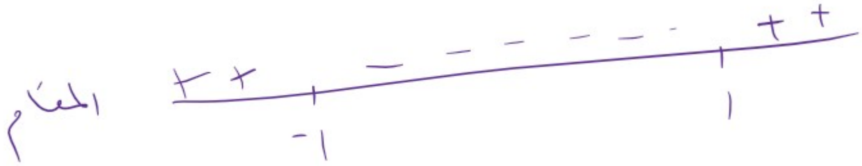
or

or if $\lim_{x\to a^-} f(x) = \infty$ or $-\infty$ ✓

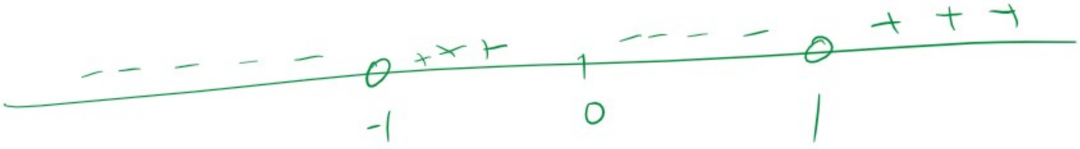
$f(x)=\frac{x}{x^2-1}$ $\Rightarrow$ $f' = \frac{(x^2-1)(1)}{\quad}$

$x=1$
$x=-1$





$f = \frac{x}{x^2-1}$



$\lim_{x\to1^+} = \infty$

$x\to1^- \quad = \infty$

$f(x) \Rightarrow f' =$

D(f)

Extrem Values ✓

تحدث عند

أو عند — End points

وإما عند — critical points ✓

$f(x) = x^{\frac{1}{3}}$ on $\underline{[0, 1]}$ D(f)

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3}\frac{1}{\sqrt[3]{x^2}}$

critical point ① $f' = 0$ or

f' undefined

f' undefined at $\boxed{x=0}$

② D(f) موجودة في

$(0, f(0))$ critical point

$\boxed{x=0}$ ✓

Th // f cont. on [a, b] then f has A. max and A. min

مهمة

1. lecture note on itc ✓ — مشكلة
   - ✓ go video
2. Discussion problem → on itc قبل الاطلاع ع الإجابة
3. Discussion " in out line ch4, ch4 ... ch1 ...

3) Discussion ⁄⁄ in out line ch4, ch7 ...

4) Previous Exames

Q1 f $\qquad f(x) = \frac{x}{x^2-1}$ $\qquad \frac{\overset{①}{x}}{\underset{②}{x}}$ $\Rightarrow \nexists$ O.Asy $\Rightarrow \exists$ H.Asy

- D(f) $\Rightarrow x^2-1=0 \Rightarrow x=\pm 1 \Rightarrow D(f) = \mathbb{R}\setminus\{\pm 1\}$

- Asy. $\nexists$ O.Asy.

$\exists$ H.Asy $\rightarrow \lim\limits_{x\to\infty} \frac{x}{x^2-1} = 0 \Rightarrow \boxed{y=0}$ is H.Asy

$\rightarrow \lim\limits_{x\to-\infty} \frac{x}{x^2-1} = 0$

V.Asy $\Rightarrow$ check zeros of denominator $\Rightarrow$ check $x=1$ ...(A)
check $x=-1$ ...(B)

A) check $\boxed{x=1} \Rightarrow \lim\limits_{x\to1^+} f(x) = \lim\limits_{x\to1^+} \frac{x}{x^2-1} = \frac{1}{\text{small}+} = +\infty$

$\Downarrow$

$\boxed{x=1}$ is V. Asy

0.9999 1.0001

1 x

$\lim\limits_{x\to1^-} f(x) = \lim\limits_{x\to1^-} \frac{x}{x^2-1} = \frac{1}{\text{small}-} = -\infty$

B) check $x=-1 \Rightarrow \lim\limits_{x\to-1^+} f(x) = \lim\limits_{x\to-1^+} \frac{x}{x^2-1} = \frac{-1}{\text{small}-} = +\infty$

$\Downarrow$

$\boxed{x=-1}$ is V.Asy

-1.0001 -0.999

-1 x

$\lim\limits_{x\to-1^-} f(x) = \lim\limits_{x\to-1^-} \frac{x}{x^2-1} = \frac{-1}{\text{small}+} = -\infty$

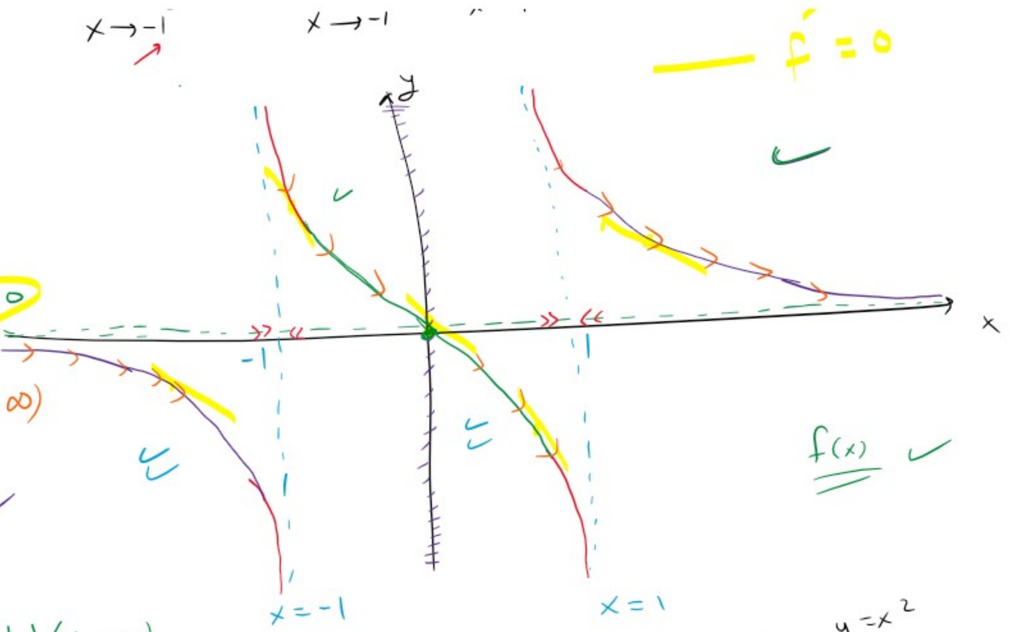
$f' = 0$

C. x xy

$f(x) = \frac{x}{x^2 - 1}$

Keypoint $(0,0)$ ✓

$x = 0 \Rightarrow y = \frac{0}{0^2 - 1} = \frac{0}{-1} = 0$

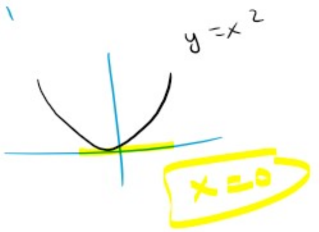


$f \downarrow$ on $(-\infty,-1) \cup (-1,1) \cup (1,\infty)$

f is not increasing ✓

f is concave up on $(-1,0] \cup (1,\infty)$

f 〃 〃 down on $(-\infty,-1) \cup [0,1)$



$R(f) = \mathbb{R} = (-\infty, \infty)$

Exp $f(x) = x^{\frac{1}{3}}$ on $[0,1]$

Find critical points

$f' = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3} \frac{1}{\sqrt[3]{x^2}}$

f' is undefined at

$\boxed{x=0} \in [0,1]$ ✓

$x=0$ is critical point

$(0, f(0)) = (0,0)$

Critical point

⇓

$\boxed{\text{نقطة موجودة داخل } D(f)}$ → out

وتحدث عندما

$f' = 0$ ✗

f' undefined ✓

$f'(\pm 1)$ undefined

Extrem Values

L. Max ⟵ A. Max

L. Min ⟵ A. Min

⇓

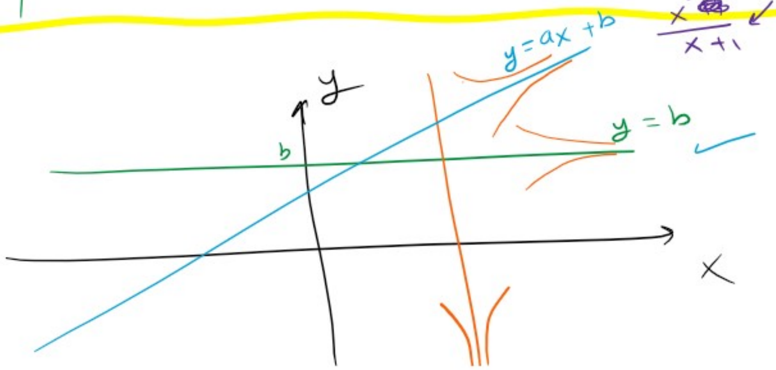
تحدث القيم القصوى عند

Critical points ✓

End Points ✓

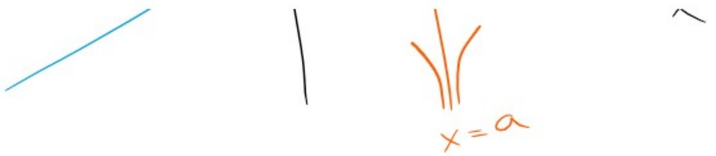
Asy. : Lines

H. V. O.



$\frac{x^2}{x+1}$

$\frac{x}{x^2+1}$



$$\frac{x}{x^2-1}$$

O. As.

$$R(x) = \frac{f(x)}{g(x)}$$

deg f is one greater than deg g

$$\underbrace{\boxed{ax+b}}_{\text{ناتج القسمة}}^{\text{O. Asy.}} + \frac{\text{الباقي}}{g(x)}$$

$2x-1$

$\frac{x}{2}+4$

x

$x+1$

Exp $f(x) = \begin{cases} x & , \quad 0 \leq x < 1 \\ 0 & , \quad x = 1 \end{cases}$

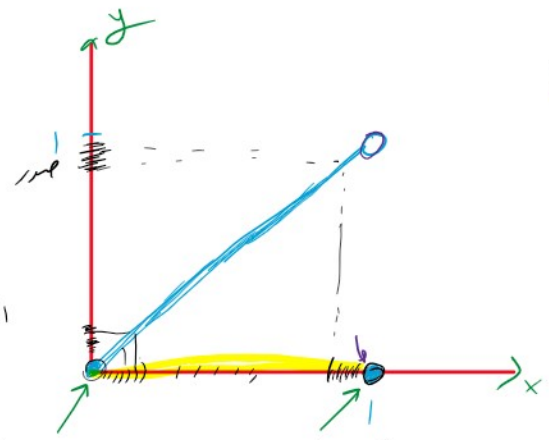
Find EV's

$\boxed{\text{[illegible]}}$ ✓

~~L. Max~~

$f(0) = 0$

$f(1) = 0$



f has L. Min at $\underline{\underline{x=0}}$ ⇒ أصغر 0 ✓
A. Min

L. Min $x=1$ ✓
A. Min

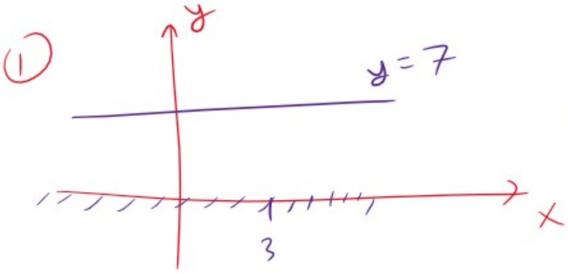
$x = \phi \Rightarrow$ ⓪

أقل صورة

*

> f(x)

Exp ①



any x=3 is Abs Max

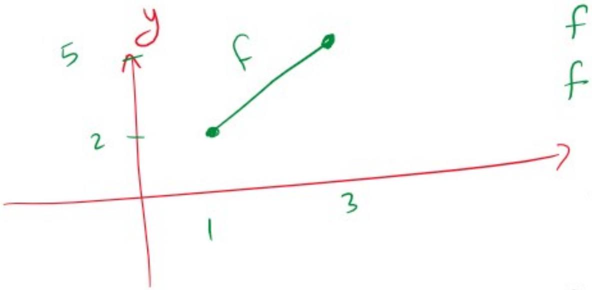
any x → Abs Min

$\Rightarrow f(3) = 7 \geq f(x)$

$f(3) = 7 \leq f(x)$

$\forall x$

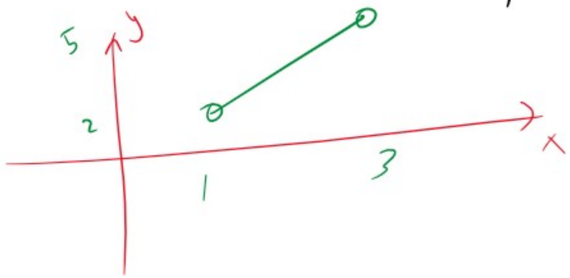
②



f has A.Max of 5 at x = 3

f = = Min of 2 = = 1

③



f has no EV's

Q1 ⓓ $f(x) = \dfrac{x^2 - 3}{x - 2}$

$\boxed{D(f) = \mathbb{R} \setminus \{2\}}$

$$\begin{array}{r|l} & x + 2 \\ \hline x - 2 & x^2 - 3 \\ & \cancel{-x^2} + 2x \\ \hline & 2x - 3 \\ & \cancel{-2x} + 4 \\ \hline & 1 \end{array}$$

∃ O.Asy

$= \boxed{x + 2} + \dfrac{1}{x - 2}$

$y = x + 2$ O.Asy.

$\Rightarrow \cancel{\exists}$ H.Asy

V.Asy Check $x = 2$ $\Rightarrow \lim\limits_{x \to 2^+} \dfrac{x^2 - 3}{x - 2} = \dfrac{4 - 3}{\text{small} +} = \boxed{+\infty}$

$\Downarrow$

$x = 2$ V.Asy

$\lim\limits_{x \to 2^-} \dfrac{x^2 - 3}{x - 2} = \dfrac{4 - 3}{\text{small} -} = -\infty$ $\Uparrow$

Keypoint $\Rightarrow f' = 0$

Max

Keypoint

$f(x) = \boxed{\frac{x^2-3}{x-2}}$

$f = 0 \Rightarrow x^2-3=0$

$x = \pm\sqrt{3}$

$f(0) = \frac{0-3}{0-2} = \frac{3}{2}$

$\Rightarrow f' = 0$

$f' = \frac{(x-2)(2x)-(x^2-3)(1)}{(x-2)^2} = 0$

$(x-2)(2x) - x^2+3=0$

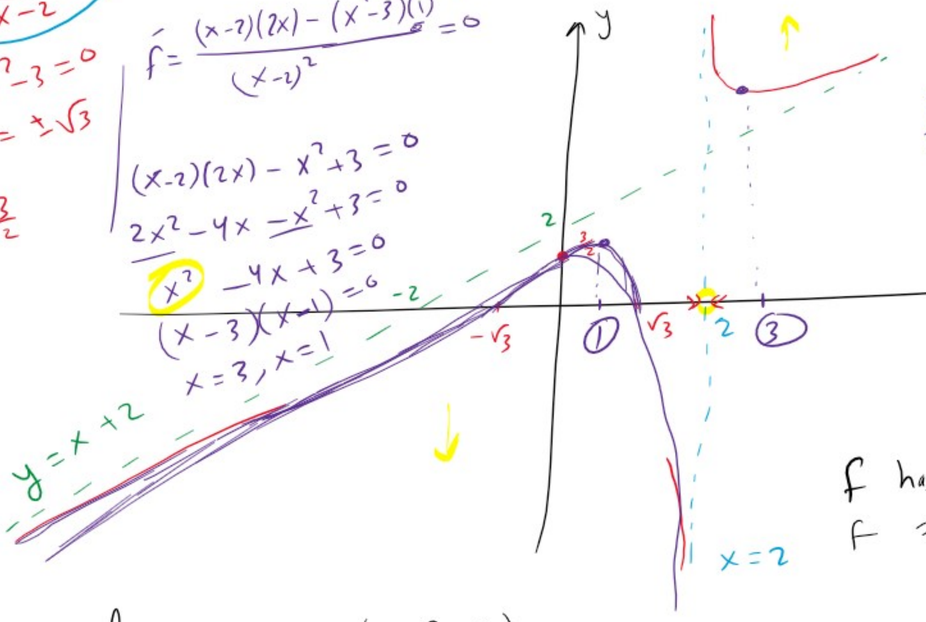
$2x^2-4x-x^2+3=0$

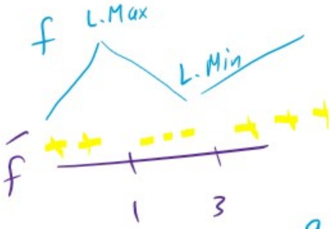
$x^2-4x+3=0$

$(x-3)(x-1)=0$

$x=3, x=1$

$\lim_{x\to 2^-} \frac{x^2-3}{x-2} = \frac{4-3}{\text{small } -} = -\infty$





$f(3) = \frac{9-3}{3-2}$

$= 6$

$f(1) = \frac{1-3}{1-2} = 2$

f has L.Max of 2 at x = 1

f " " Min of 6 at x = 3

f concave down on $(-\infty, 2)$

" " up " $(2, \infty)$

"