

Q2 (f) $y = f(x) = \frac{x}{x^2 - 1}$

$\frac{x}{x^0} \Rightarrow \exists$ H. Asy.
 $\Rightarrow \nexists$ O. Asy.

$D(f) \Rightarrow x^2 - 1 = 0 \Rightarrow \underline{x = \pm 1} \Rightarrow \underline{D(f) = \mathbb{R} \setminus \{\pm 1\}}$

• Asy: $\nexists$ O. Asy.

• H. Asy: $\checkmark \lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{x}{x^2 - 1} = 0 \Rightarrow y = 0$ H. Asy.

$\checkmark \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{x}{x^2 - 1} = 0$

• V. Asy $\Rightarrow$ check zeros for denominator ① check $x = -1$
 ③ check $x = 1$

① $x = -1 \Rightarrow \lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} \frac{x}{x^2 - 1} = \frac{-1}{\text{small } -} = +\infty$
 $\Rightarrow x = -1$ is V. Asy.

$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} \frac{x}{x^2 - 1} = \frac{-1}{\text{small } +} = -\infty$

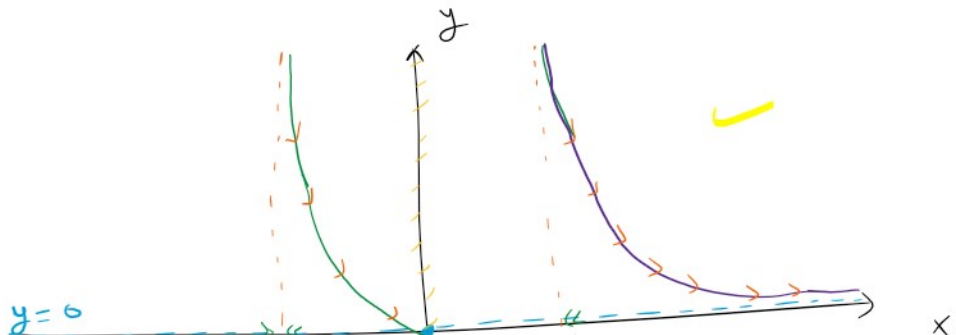
② $x = 1 \Rightarrow \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{x}{x^2 - 1} = \frac{1}{\text{small } +} = +\infty$
 $x = 1$ is V. Asy

$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \frac{x}{x^2 - 1} = \frac{1}{\text{small } -} = -\infty$

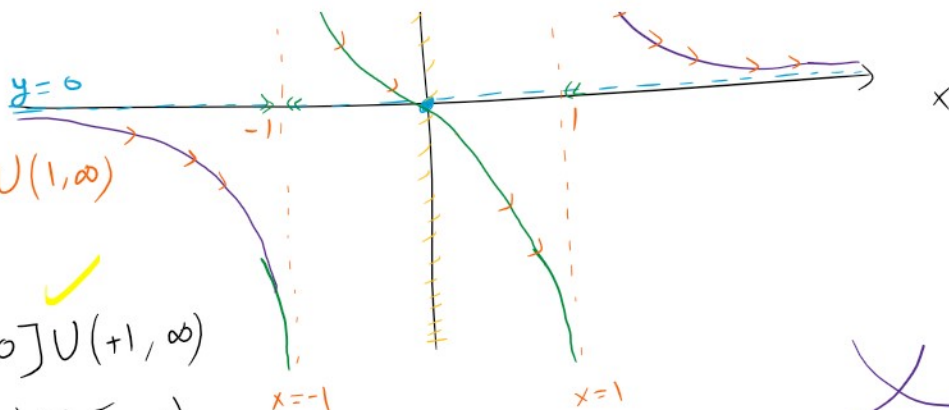
$f(x) = \frac{x}{x^2 - 1}$

Key point (0,0)

f is not increasing



f is not increasing

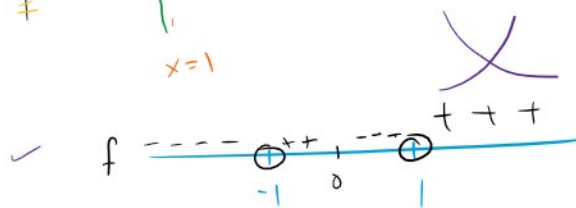


$f \downarrow$ on $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

f concave up on $(-1, 0] \cup (1, \infty)$

f concave down on $(-\infty, -1) \cup [0, 1)$

Range = $\mathbb{R} = (-\infty, \infty)$

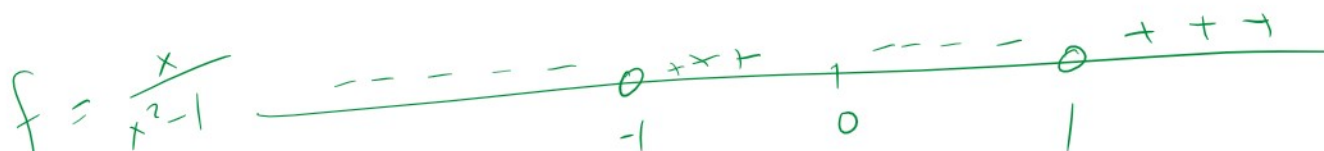
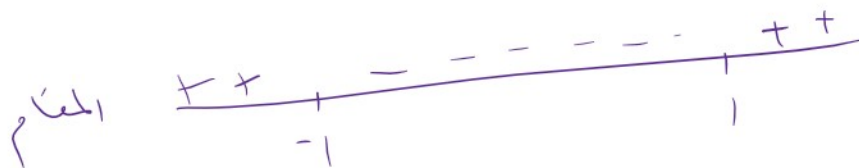


$x=a$ is V. Asy. if $\lim_{x \rightarrow a^+} f(x) = \infty$ or $-\infty$ ✓

or if $\lim_{x \rightarrow a^-} f(x) = \infty$ or $-\infty$ ✓

$$f(x) = \frac{x}{x^2-1} \Rightarrow \bar{f} = \frac{(x^2-1)(1)}{x^2-1}$$

$$\begin{matrix} x=1 \\ x=-1 \end{matrix}$$



$\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$
 $\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$

$$f(x) \Rightarrow f' =$$

Diagram illustrating the relationship between a function $D(f)$ and its extremal values:

- The function $D(f)$ is shown as a curve.
- The left side of the curve is labeled "A. Max" (Absolute Maximum) and "L. Max" (Local Maximum).
- The right side of the curve is labeled "L. Min" (Local Minimum) and "A. Min" (Absolute Minimum).
- A purple oval at the bottom contains the text "Extrem Values" with a green checkmark and a blue arrow pointing to the curve.

A hand-drawn diagram illustrating the components of a function's domain. A blue curve is shown with a purple line segment above it. The curve starts at a point labeled 'End points' and ends at a point labeled 'critical points'. A bracket connects these two points, with the word 'interval' written above it. A checkmark is at the bottom right.

$f(x) = x^{\frac{1}{3}}$ on $\underline{\underline{[0, 1]}}$
 $D(f)$

$$f(x) = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3} \frac{1}{\sqrt[3]{x^2}}$$

critical point $\begin{cases} \textcircled{1} f' = 0 \text{ or} \\ f \text{ undefined} \end{cases}$

f' undefined at $x=0$

✓ 2 معلومة 3 $D(f)$

$(0, f(0))$ critical point
 $x=0$ ✓

Th f cont. on $[a, b]$ then f has A. max and A. min

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① lecture note on itc ✓

✓ go video

② Discussion problem ^{on it} $\rightarrow$ $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

[3] Discussion : in out line $\text{CH}_3\text{---CH}_2\text{CH}_2\text{---}$

3) Discussion in outline ch4, ch7...

4) Previous Exams

Q1 $f(x) = \frac{x}{x^2 - 1}$ $\frac{0}{0} \Rightarrow \cancel{\text{O. Asy}} \Rightarrow \exists \text{ H. Asy}$

• $D(f) \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1 \Rightarrow D(f) = \mathbb{R} \setminus \{\pm 1\}$

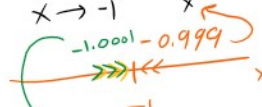
• Asy. $\cancel{\text{O. Asy.}}$
 $\exists \text{ H. Asy} \rightarrow \lim_{x \rightarrow \infty} \frac{x}{x^2 - 1} = 0 \Rightarrow y = 0 \text{ is H. Asy}$
 $\rightarrow \lim_{x \rightarrow -\infty} \frac{x}{x^2 - 1} = 0$

V. Asy $\Rightarrow$ check zeros of denominator $\Rightarrow$ check $x = 1$... (A)
 check $x = -1$... (B)

A) check $x = 1 \Rightarrow \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{x}{x^2 - 1} = \frac{1}{\text{small}^+} = +\infty$
 $x = 1$ is V. Asy
 $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \frac{x}{x^2 - 1} = \frac{1}{\text{small}^-} = -\infty$



B) check $x = -1 \Rightarrow \lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} \frac{x}{x^2 - 1} = \frac{-1}{\text{small}^-} = +\infty$
 $x = -1$ is V. Asy
 $\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} \frac{x}{x^2 - 1} = \frac{-1}{\text{small}^+} = -\infty$



$f' = 0$

$$f(x) = \frac{x}{x^2 - 1}$$

key point $(0,0)$ ✓

$$x=0 \Rightarrow y = \frac{0}{0-1} = \frac{0}{-1} = 0$$

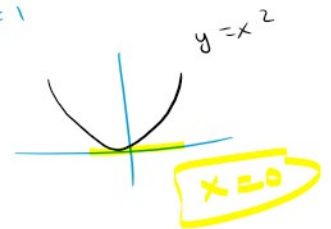
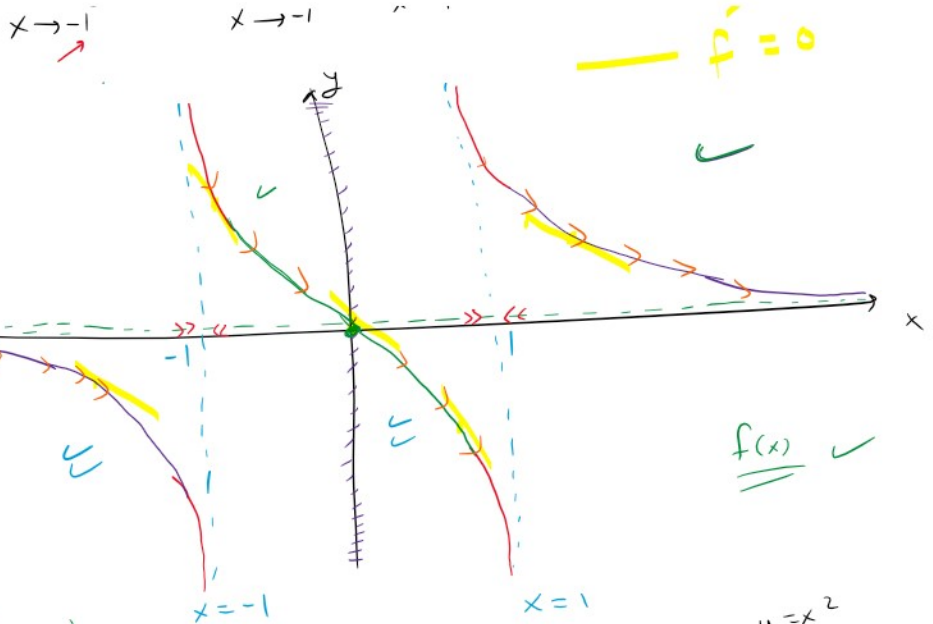
$f \downarrow$ on $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

f is not increasing ✓

f is concave up on $(-1, 0] \cup (1, \infty)$

f " " down on $(-\infty, -1) \cup [0, 1)$

$$R(f) = \mathbb{R} = (-\infty, \infty)$$



Exp $f(x) = x^{\frac{1}{3}}$ on $[0,1]$

Find critical points

$$f' = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3} \frac{1}{\sqrt[3]{x^2}}$$

f' is undefined at

$$x=0 \in [0,1]$$

$x=0$ is critical point
 $(0, f(0)) = (0,0)$

Critical point

نقطة حرجية
 $D(f)$ داخل
وغيره خارج

$$f' = 0$$

f' undefined

$f(1)$ undefined

Extrem Values

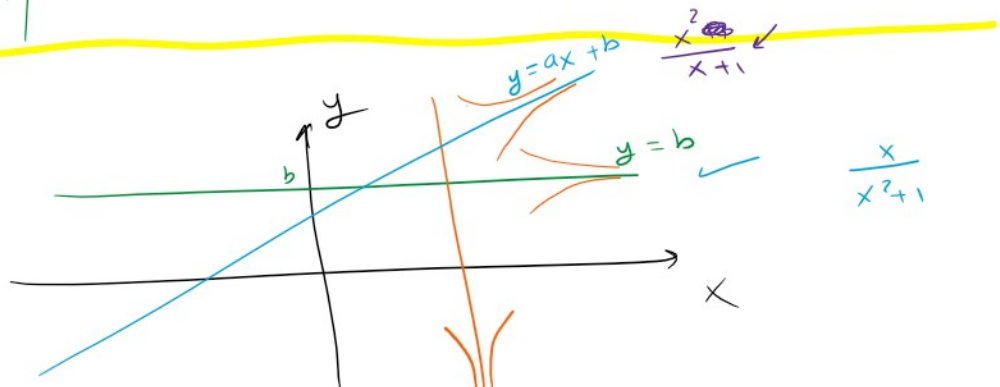
L. Max $\leftarrow$ A. Max
L. Min $\leftarrow$ A. Min

نقطة القيم القصوى

Critical points

End Points

Asy: Lines
H. v. 0.



$$x=a$$

$$\frac{x}{x^2-1}$$

O. Asy.

$$R(x) = \frac{f(x)}{g(x)}$$

deg f is one greater than deg g

$$\boxed{\text{O. Asy.}} \quad \boxed{ax+b} + \frac{\text{f.w.}}{g(x)}$$

$$\begin{array}{c} 2x-1 \\ \frac{x}{2} + 1 \\ x \\ x+1 \end{array}$$

Exp $f(x) = \begin{cases} x & , 0 \leq x < 1 \\ 0 & , x = 1 \end{cases}$

Find EV's

EV's

~~L. Max~~



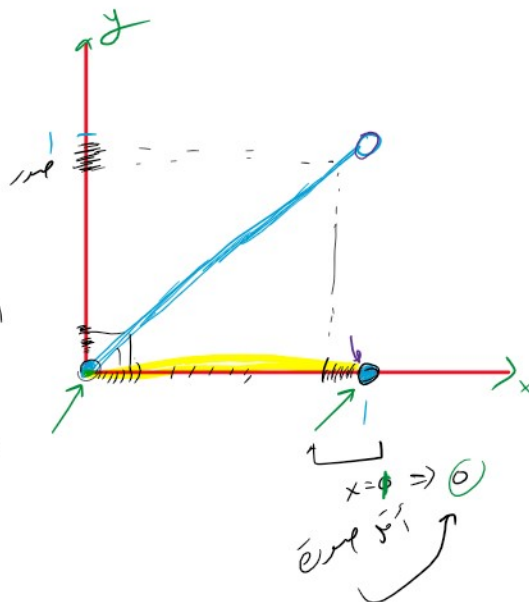
$$f(0) = 0$$

$$f(1) = 0$$

f has L. Min at $x=0$ $\Rightarrow$ 0 $\Rightarrow$ 0

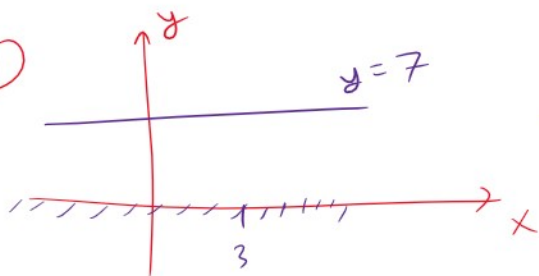
L. Min
A. Min

$$x=1$$



$x=0 \Rightarrow 0$
 $\Rightarrow$ 0

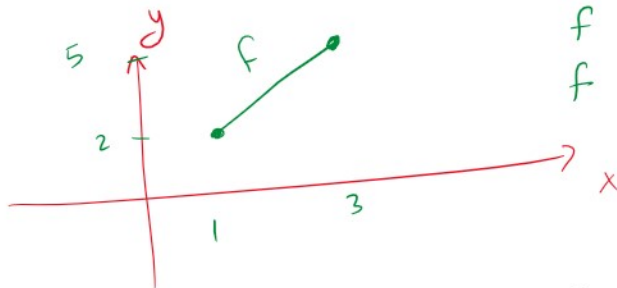
Exp ①



any $x=3$ is Abs Max
any x is Abs Min

$$\Rightarrow f(3) = 7 \geq f(x) \\ f(3) = 7 \leq f(x) \\ \forall x$$

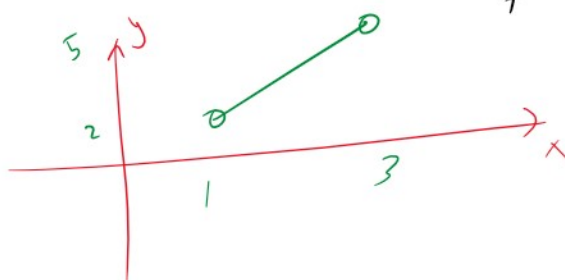
②



f has A. Max of 5 at $x=3$
 f = = Min of 2 = = 1

f has no EU's

③



Q1 d $f(x) = \frac{x^2 - 3}{x - 2}$

$D(f) = \mathbb{R} \setminus \{2\}$

$\exists$ O. Asy

$= x + 2 + \frac{1}{x - 2}$

$y = x + 2$ O. Asy.

$$\begin{array}{r} x+2 \\ x-2 \overline{) x^2-3} \\ \underline{+x^2+2x} \\ 2x-3 \\ \underline{-2x+4} \\ 1 \end{array}$$

$\Rightarrow$ ~~7~~ H. Asy.

V. Asy

check

$x=2$

$\Rightarrow$

$\lim_{x \rightarrow 2^+} \frac{x^2 - 3}{x - 2} = \frac{4 - 3}{\text{small}^+} = +\infty$

$x=2$

V. Asy

Key point

$\frac{0}{0} \rightarrow f' = 0$

$\lim_{x \rightarrow 2^-} \frac{x^2 - 3}{x - 2} = \frac{4 - 3}{\text{small}^-} = -\infty$

Key point
 $f(x) = \frac{x^2 - 3}{x - 2}$

$$f(x) = \frac{x^2 - 3}{x - 2}$$

$$f=0 \Rightarrow x^2-3=0$$
$$x = \pm\sqrt{3}$$

$$f(0) = \frac{0-3}{0-2} = \frac{3}{2}$$

$$\Rightarrow f = 0$$

$$f' = \frac{(x-2)(2x) - (x^2-3)(1)}{(x-2)^2} = 0$$

$$(x-2)(2x) - x^2 + 3 = 0$$

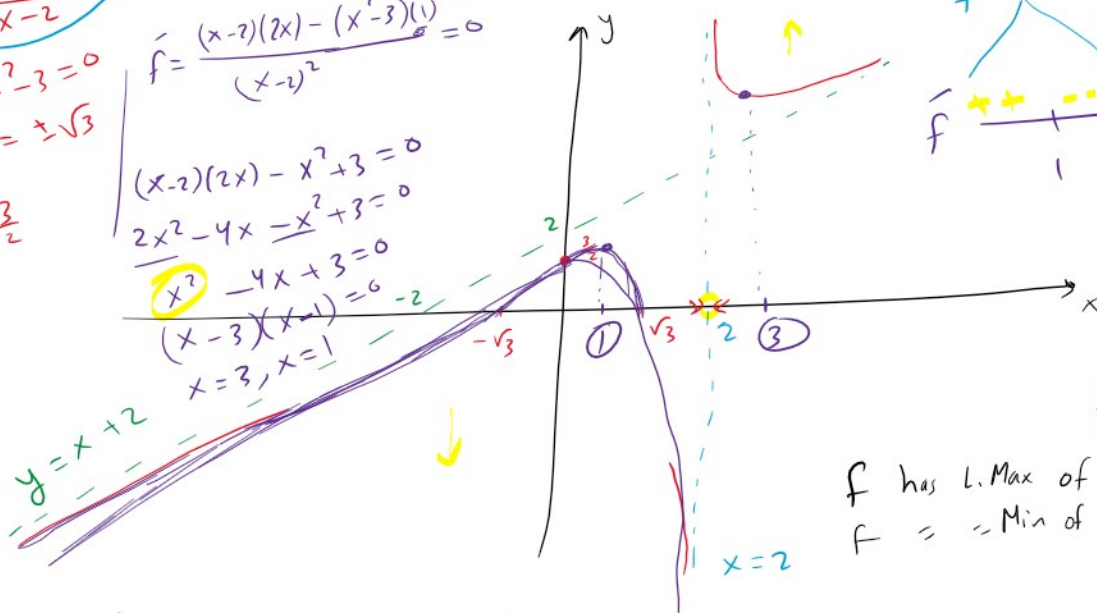
$$2x^2 - 4x - \frac{x^2}{-1} + 3 = 0$$

$$\frac{2x^2 - 9x}{x^2} - 4x + 3 = 0$$

$$(x-3)(x-1) = 6$$

$$x = 3, x = 1$$

$$\lim_{x \rightarrow 2^-} \frac{x^2 - 3}{x - 2} = \frac{4 - 3}{\text{small } -} = -\infty$$



$$f(3) = \frac{9-3}{3-2} = 6$$

$$f(1) = \frac{1-3}{1-2} = 2$$

f has L. Max of 2 at $x=1$
 f = - Min of 6 at $x=3$

f concave down on $(-\infty, 2)$
 " " up " $(2, \infty)$