$Y_1 \sim \text{Poisson}(\mu_1 + \mu_2)$

$$\left.\begin{array}{l} X_1 \sim \text{Poisson}(\mu_1) \\ X_2 \sim \text{Poisson}(\mu_2) \\ X_1, X_2 \text{ indep.} \end{array}\right\} \Rightarrow X_1 + X_2 \sim \text{Poisson}(\mu_1 + \mu_2).$$

Note : Binomial formula

$$(a+b)^n = \sum_{m=0}^{n} \binom{n}{m} a^m b^{n-m}$$

Exercises.

Q17: let X have a p.d.f $f(x) = \begin{cases} \frac{1}{3} & , x = 1, 2, 3. \\ 0 & , \text{elsewhere} \end{cases}$

Find the p.d.f of $y = 2X+1$.

$y = 2X+1 \rightarrow X = \frac{y-1}{2}$

$U : A \to B$, $A = 1, 2, 3$

$B = 3, 5, 7$.

$$\Rightarrow g(y) = \begin{cases} f\left(\frac{y-1}{2}\right) & , y = 3, 5, 7 \\ 0 & , \text{elsewhere} \end{cases}$$

$$= \begin{cases} \frac{1}{3} & , y = 3, 5, 7 \\ 0 & , \text{elsewhere} \end{cases}$$

Q2o: let X_1, X_2 have the joint p.d.f, $f(x_1, x_2) = \begin{cases} \frac{x_1 x_2}{36} & , x_1 = 1,2,3 , x_2 = 1,2,3 \\ 0 & , \text{elsewhere} \end{cases}$

Find the joint p.d.f of $y_1 = x_1 \cdot x_2$ and $y_2 = x_2$ and then find the marginal p.d.f of y_1.

$y_1 = x_1 \cdot y_2 \rightarrow x_1 = \frac{y_1}{y_2}$ $x_2 = y_2$

$U: A \rightarrow B$

$A: \{(x_1, x_2) : x_1 = 1,2,3 , x_2 = 1,2,3\}$.

$B: \{(y_1, y_2) : y_1 = 1,2,3,4,6,9 , y_2 = 1,2,3\}$

x_1	x_2	y_1	y_2	$f(x_1, x_2)$
1	1	1	1	$\frac{1}{36}$
1	2	2	2	$\frac{2}{36}$
1	3	3	3	$\frac{3}{36}$
2	1	2	1	$\frac{2}{36}$
2	2	4	2	$\frac{4}{36}$
2	3	6	3	$\frac{6}{36}$
3	1	3	1	$\frac{3}{36}$
3	2	6	2	$\frac{6}{36}$
3	3	9	3	$\frac{9}{36}$

$$\rightarrow g(y_1, y_2) = \begin{cases} f(w_1(y_1, y_2), w_2(y_1, y_2)) , & (y_1, y_2) \in B \\ 0 , & \text{else} \end{cases}$$

$$= \begin{cases} f(\frac{y_1}{y_2}, y_2) , & (y_1, y_2) \in B \\ 0 , & \text{else} \end{cases}$$

$$= \begin{cases} \frac{\frac{y_1}{y_2} \cdot y_2}{36} , & (y_1, y_2) \in B \\ 0 , & \text{else} \end{cases}$$

$$g(y_1, y_2) = \begin{cases} \frac{y_1}{36} , & (y_1, y_2) \in B \\ 0 , & \text{else} \end{cases}$$

$$g(y_1, y_2) = \begin{cases} \frac{1}{36} , & (y_1, y_2) = (1,1) \\ \frac{2}{36} , & (y_1, y_2) = (2,2) , (2,1) \\ \frac{3}{36} , & (y_1, y_2) = (3,3) , (3,1) \\ \frac{4}{36} , & (y_1, y_2) = (4,2) \\ \frac{6}{36} , & (y_1, y_2) = (6,3) , (6,2) \\ \frac{9}{36} , & (y_1, y_2) = (9,3) \end{cases}$$

So $g_1(y_1) = \begin{cases} \frac{1}{36} & , y_1 = 1 \\ \frac{2}{36} + \frac{2}{36} = \frac{4}{36} & , y_1 = 2 \\ \frac{3}{36} + \frac{3}{36} = \frac{6}{36} & , y_1 = 3 \\ \frac{4}{36} & , y_1 = 4 \\ \frac{6}{36} + \frac{6}{36} = \frac{12}{36} & , y_1 = 6 \\ \frac{9}{36} & , y_1 = 9 \end{cases}$

Note:

$A = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

$B = \{(1,1),(2,2),(3,3),(2,1),(4,2),(6,3),(3,1),(6,2),(9,3)\}$.

Q21 ✓

4.2:

Q21: let the independent random variables X_1 and X_2 be $b(n_1, P)$ and $b(n_2, P)$, respectively Find the joint p.d.f of $y_1 = X_1 + X_2$ and $y_2 = X_2$ and then find the marginal p.d.f of y_1.

$$X_1 \sim b(n_1, P) \Rightarrow f(x_1) = \begin{cases} \binom{n_1}{x_1} P^{x_1} (1-P)^{n_1 - x_1}, & x_1 = 0, 1, \ldots, n_1 \\ 0, & \text{elsewhere} \end{cases}$$

$$X_2 \sim b(n_2, P) \Rightarrow f(x_2) = \begin{cases} \binom{n_2}{x_2} P^{x_2} (1-P)^{n_2 - x_2}, & x_2 = 0, 1, \ldots, n_2 \\ 0, & \text{elsewhere} \end{cases}$$

Since X_1, X_2 indep. $f(x_1, x_2) = \begin{cases} \binom{n_1}{x_1} P^{x_1} (1-P)^{n_1 - x_1} \cdot \binom{n_2}{x_2} P^{x_2} (1-P)^{n_2 - x_2}, & (x_1, x_2) \in A \\ 0, & \text{elsewhere} \end{cases}$

$Y_1 = X_1 + X_2 \to X_1 = y_1 - y_2$

$Y_2 = X_2 \to X_2 = y_2$.

$A = \{(x_1, x_2) : x_1 = 1, \ldots, n_1, \; x_2 = 1, \ldots, n_2\}$

$B = \{(y_1, y_2) : y_1 = 1, \ldots, n_1 + n_2, \; y_2 = 1, \ldots, y_1\}$

$$\Rightarrow g(y_1, y_2) = \begin{cases} f(y_1 - y_2, y_2), & (y_1, y_2) \in B \\ 0, & \text{elsewhere} \end{cases}$$

$$= \begin{cases} \binom{n_1}{y_1 - y_2} P^{y_1 - y_2} (1-P)^{n_1 - (y_1 - y_2)} \cdot \binom{n_2}{y_2} P^{y_2} (1-P)^{n_2 - y_2}, & (y_1, y_2) \in B \\ 0, & \text{else} \end{cases}$$

$n_1 - y_1 + \cancel{y_2} + n_2 - \cancel{y_2}$

Now $g_1(y_1) = \sum_{y_2} g(y_1, y_2) = \sum_{y_2=0}^{y_1} \binom{n_1}{y_1 - y_2} \underline{p}^{y_1 - y_2} (1-p)^{n_1 - (y_1 - y_2)} \cdot \binom{n_2}{y_2} \underline{p}^{y_2} (1-p)^{n_2 - y_2}$

$$= \underbrace{\sum_{y_2=0}^{y_1} \binom{n_1}{y_1 - y_2}\binom{n_2}{y_2}} p^{y_1} (1-p)^{n_1+n_2-y_1}$$

$$g_1(y_1) = \binom{n_1+n_2}{y_1} p^{y_1} (1-p)^{n_1+n_2-y_1} \quad , y_1 = 0, 1, \dots, n_1+n_2$$

$y_1 \sim b(n_1+n_2, p)$.