

Exponential Function

$y = e^x$ is positive function

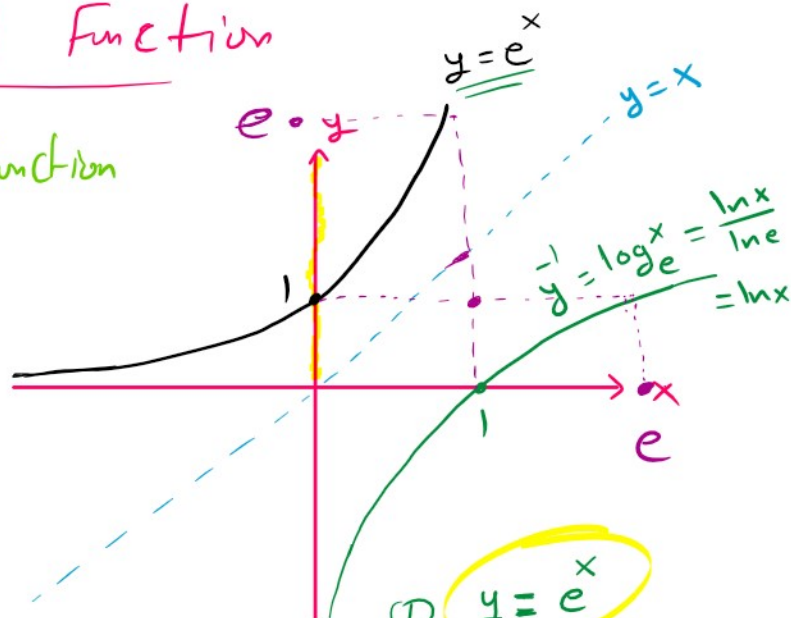
$$D = (-\infty, \infty)$$

$$R = (0, \infty)$$

$$\lim_{x \rightarrow \infty} e^x = \infty$$

$$\lim_{x \rightarrow -\infty} e^x = 0$$

x	y
$\frac{1}{2}$	$e^{\frac{1}{2}}$
0	$e^0 = 1$
1	e
2	e^2



① $y = e^x$

② $\ln y = \ln e^x$

$$\ln y = x (\ln e)$$

• $\ln y = x$

③ $y = \ln x$

④ $f^{-1}(x) = \ln x$

Properties for e^x

① $e^{\ln x} = x \quad \forall \underline{x > 0}$

$$e^{\ln 3} = 3, \quad e^{\ln \sqrt{2}} = \sqrt{2}, \quad e^{\ln \frac{1}{2}} = \frac{1}{2}$$

② $\ln e^x = x \quad \forall \underline{x}$

$$x \ln e = x(1) = x$$

$$+ \Rightarrow e^{-2} = \frac{1}{e^2}$$

$$\ln e^3 = 3, \quad \ln e^{-2} = -2$$

Exp Solve for x (1) $\ln e^{2x} = 10$

$$2x = 10$$

$$x = 5$$

(2) $(\ln 2)^x e = 10$

$$\ln e^{(\ln 2)x} = \ln 10$$

$$\frac{(\ln 2)x}{\ln 2} = \frac{\ln 10}{\ln 2}$$

$$x = \frac{\ln 10}{\ln 2}$$

Q. How to derive exponential function
 $u(x)$

A: If $y = a^{u(x)}$ where $u(x)$ diff $a > 0$

$$y' = u'(x) a^{u(x)} \ln a$$

Exp ① $y = x^3 \Rightarrow y' = 3x^2$

$\Rightarrow$ ② $y = 3^{x^2} \Rightarrow y' = 2x \cdot 3^{x^2} \ln 3$

$$y = a^{u(x)} \Rightarrow y' = a^{u(x)} u'(x) \ln a$$

$$\int a^{u(x)} u'(x) \ln a \, dx = a^{u(x)} + C$$

Exp Find y' if ① $y = e^{x^3}$

$$y' = e^{x^3} (3x^2) \ln e$$

$$\begin{aligned} \dot{y} &= e^{x^3} (3x^2) \underline{\underline{\ln e}} \\ &= 3x^2 e^{x^3} \end{aligned}$$

② $y = e^{3-5x^2}$ Find $\dot{y}(0)$

$$\begin{aligned} \dot{y} &= e^{3-5x^2} (-10x) \ln e \\ &= -10x e^{3-5x^2} \end{aligned}$$

$$\dot{y}(0) = -10(0) e^3 = 0$$

③ $y = \frac{\sin x}{\pi}$ Find $\dot{y}(\pi)$

$$\dot{y} = \frac{\sin x}{\pi} (\cos x) \ln \pi$$

$$\begin{aligned} \dot{y}(\pi) &= \frac{\sin \pi}{\pi} (\cos \pi) \ln \pi \\ &= \frac{0}{\pi} (-1) \ln \pi \end{aligned}$$

$$= 1 \quad (-1) \quad \ln \pi$$

$$= -\ln \pi$$

$$= \ln \frac{1}{\pi}$$

$$= \ln \frac{1}{\pi}$$

$$\begin{aligned} y &= e^x \\ y' &= e^x \quad (1) \quad (1) \\ y' &= e^x \end{aligned}$$

Exp Find ① $\int_{\ln 3}^{\ln 4} e^x dx$

$$\begin{aligned} &= e^x \Big|_{\ln 3}^{\ln 4} = \ln 4 \quad \ln 3 \\ &= e^{\ln 4} - e^{\ln 3} \\ &= 4 - 3 \\ &= 1 \end{aligned}$$

② $\int_0^1 \underline{3t^2} \underline{e^{t^3}} \underline{dt}$

$$\begin{aligned} u &= t^3 \\ du &= 3t^2 dt \end{aligned}$$

$$\begin{aligned} \int_0^1 e^u du &= e^u \Big|_0^1 = e^1 - e^0 \\ &= e - 1 \end{aligned}$$

$$t=0 \Rightarrow u = 0^3 = 0$$

$$t=0 \Rightarrow u=0=0$$

$$t=1 \Rightarrow u=1=1$$



(3) $\int_0^{\frac{\pi}{3}} \frac{\sec \theta}{5 \ln 5} \sec \theta \tan \theta d\theta$

$$u = \sec \theta$$

$$du = \sec \theta \tan \theta d\theta$$

$$\int_1^2 \frac{u}{5 \ln 5} du$$

$$\left. \frac{u^2}{5} \right|_1^2 = \frac{2^2}{5} - \frac{1^2}{5}$$

$$= \frac{25}{5} - \frac{5}{5}$$

$$= 20$$

$$\theta = 0 \Rightarrow u = \sec 0 = 1$$

$$\theta = \frac{\pi}{3} \Rightarrow u = \sec \frac{\pi}{3} = \frac{1}{\frac{1}{2}} = 2$$

(4) $\int \frac{x}{3} dx$

$$= \frac{1}{\ln 3} \left(\int \frac{x}{3} \ln 3 dx \right)$$

$$= \frac{1}{\ln 3} \left(\frac{x^2}{2} \ln 3 \right) + C$$

Properties

$$\boxed{1} \quad e^{x_1} e^{x_2} = e^{x_1 + x_2}$$

$$e^{-2} e^5 = e^{-2+5} = e^3$$

$$e^{\frac{1}{2}} e^3 = e^{\frac{1}{2}+3} = e^{3.5} = e^{\frac{7}{2}}$$

$$\boxed{2} \quad \frac{e^{x_1}}{e^{x_2}} = e^{x_1 - x_2}$$

$$\frac{e^3}{e^1} = e^{3-1} = e^2$$

$$\frac{e^{\sqrt{5}}}{e^2} = e^{\sqrt{5}-2}$$

$$\boxed{3} \quad \frac{1}{e^x} = e^{-x}$$

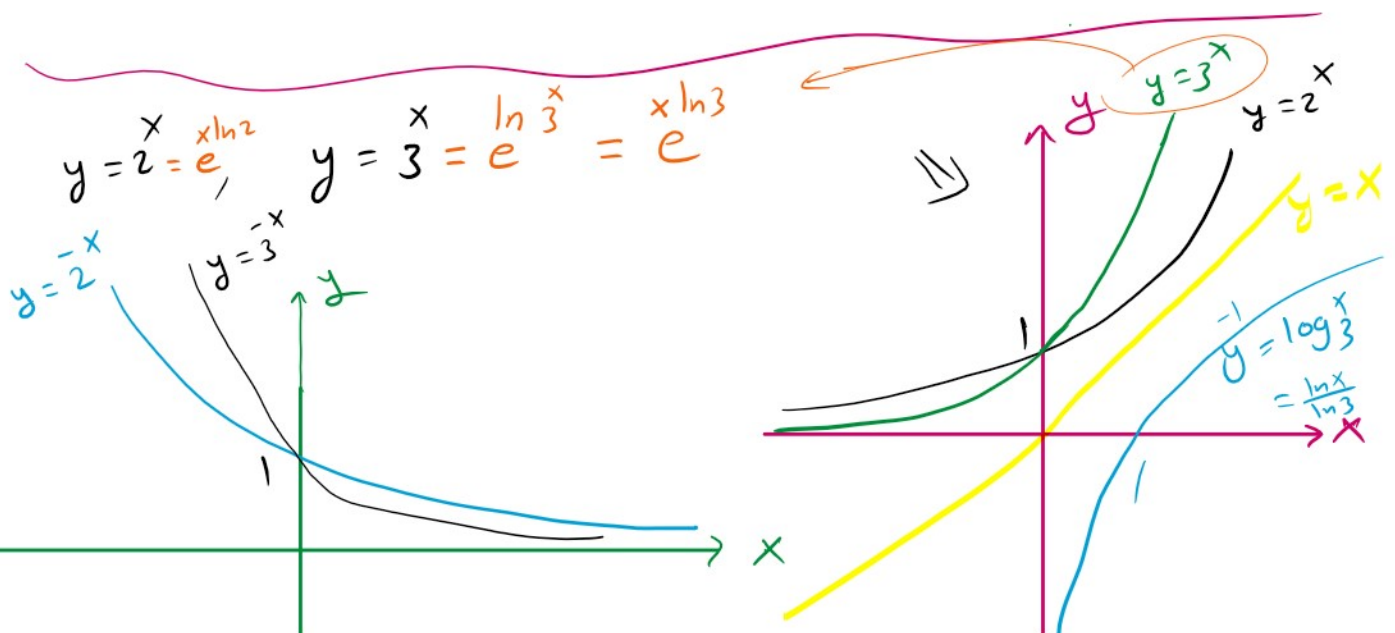
$$\boxed{3} \quad e^{-x} = \frac{1}{e^x}$$

$$e^{-2} = \frac{1}{e^2} \quad , \quad e^{-\frac{1}{2}} = \frac{1}{e^{\frac{1}{2}}} = \frac{1}{\sqrt{e}}$$

$$\boxed{4} \quad (e^{x_1})^{x_2} = e^{x_1 \cdot x_2}$$

$$(e^2)^3 = e^{2 \cdot 3} = e^6$$

$$(e^{-\frac{1}{2}})^{14} = e^{-\frac{1}{2} \cdot 14} = e^{-7} = \frac{1}{e^7}$$



$$y = 2^{-x} = e^{\ln 2^{-x}} = e^{-x \ln 2} = (2^{-1})^x = \left(\frac{1}{2}\right)^x$$

$$y = 3^{-x} = e^{\ln 3^{-x}} = e^{-x \ln 3} = (3^{-1})^x = \left(\frac{1}{3}\right)^x$$

Exp $y = x^x$ Find $y'(1)$

$$\ln y = \ln x^x$$

$$\ln y = x \ln x$$

$$\frac{y'}{y} = (x) \left(\frac{1}{x} \right) + (\ln x) (1)$$

$$= 1 + \ln x$$

$$y' = y [1 + \ln x]$$

$$y' = x^x [1 + \ln x]$$

$$y'(1) = 1^1 [1 + \ln 1]$$

$$\begin{aligned}
 y'(1) &= 1 [1 + \ln 1] \\
 &= 1 [1 + 0] \\
 &= 1
 \end{aligned}$$

Exp Find y' if (1) $y = \log_y x^2$

$$y = \frac{\ln x^2}{\ln y} = \frac{1}{\ln y} \ln x^2$$

$$y' = \frac{1}{\ln y} \cdot \frac{2x}{x^2}$$

$$= \frac{1}{\ln 2} \cdot \frac{2x}{x^2} = \frac{1}{\cancel{2} \ln 2} \cdot \frac{\cancel{2} x}{x^2} = \frac{1}{x \ln 2}$$

$$\begin{aligned}
 \textcircled{2} \quad y &= \log_2 8x^{\ln 2} \\
 &= \frac{\ln 8 \cdot x^{\ln 2}}{\ln 2} = \frac{\ln 8 + \ln x^{\ln 2}}{\ln 2}
 \end{aligned}$$

$$= \frac{\ln^3 2 + \ln 2 \ln x}{\ln 2}$$

$$= \frac{3\ln 2 + \ln 2 \ln x}{\ln 2}$$

$$= \frac{\cancel{\ln 2} [3 + \ln x]}{\cancel{\ln 2}}$$

$$y = 3 + \ln x$$

$$y' = 0 + \frac{1}{x}$$

$$= \frac{1}{x} \quad \checkmark$$

$$\textcircled{3} \quad y = \int_0^{\log_2 x} 2 \ln 2 \cdot y \, dt$$

$$y' = 2 \ln 2 \cdot y \cdot (\log_2 x)'$$

$$\begin{aligned} \Downarrow \\ 4^{\log_2 x} &= (2^2)^{\log_2 x} \\ &= 2^{2 \log_2 x} \\ &= 2^2 \\ &= 4 \end{aligned}$$

$$= 2 \ln 2 \quad y^{\log_2 x} \left(\frac{\ln x}{\ln 2} \right)$$

$$\begin{aligned} &= 2 \log_2 x^2 \\ &= x^2 \end{aligned}$$

$$= 2 \cancel{\ln 2} \quad y^{\log_2 x} \quad \frac{1}{\cancel{\ln 2}} \quad \frac{1}{x} \quad \leftarrow$$

$$= 2 \quad x^2 \quad \frac{1}{x}$$

$$= 2x$$

$$y^{\log_2 x} = x^2$$

$$\log_a f(x) = f(x)$$

$$\log_a f(x) = \ln f(x)$$

$$\log_a f(x) = \ln f(x)$$

$$\frac{\ln f(x)}{\ln a} = \ln f(x)$$

$$f(x) = \ln f(x)$$

$$\ln f(x) = \ln f(x)$$

Ex 2 If $\log_2^{4x} = 8$

Find x

$$2^8 = 4x$$

$$x = \frac{2^8}{4} = 64$$

$$\frac{\ln 4x}{\ln 2} = 8$$

$$\Rightarrow \ln 4x = 8 \ln 2$$

$$\ln(4x) = \ln 2^8$$

$$4x = 2^8$$

$$4x = 2^4 \cdot 2^4$$

$$x = \frac{16 \cdot 16}{4}$$

$$x = 64$$

$$\log_2^{4x} = 8$$

$$\log_2^{4x} = 8$$

$$4x = 2^8$$

$$\Rightarrow x = \frac{2^8}{4} = 64$$

(26) Find y if $\ln(xy) = e^{x+y}$

$$y' = \frac{dy}{dx}$$

$$\ln x + \ln y = e^{x+y}$$

$$y = \frac{-1}{x}$$

$$\ln x + \ln y = c$$

$$\frac{1}{x} + \frac{y'}{y} = e^{x+y} (1+y')$$

$$\frac{1}{x} + \frac{y'}{y} = e^{x+y} (1+y')$$

$$\frac{1}{x} + \frac{y'}{y} = e^{x+y} + y' e^{x+y}$$

$$y' \left[\frac{1}{y} - e^{x+y} \right] = e^{x+y} - \frac{1}{x}$$

$$y' = \frac{e^{x+y} - \frac{1}{x}}{\frac{1}{y} - e^{x+y}}$$

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$$\int \frac{e^{-\frac{1}{x^2}}}{x^3} dx$$

$$u = -\frac{1}{x^2} = -x^{-2}$$

$$du = 2x^{-3} dx$$

$$\int u \, du$$

$$\frac{du}{2} = \frac{dx}{x^3}$$

$$\int e^u \frac{du}{2}$$

$$\left(\frac{-1}{2}\right) = \frac{-1}{x^3}$$

$$\frac{1}{2} \int e^u du = \frac{1}{2} e^u + C$$

$$= \frac{1}{2} e^{-\frac{1}{x^2}} + C$$

(57) Find $\frac{dy}{dz}$ if $y = 5^{\sqrt{z}}$

$$y' = 5^{\sqrt{z}} \cdot \frac{1}{2\sqrt{z}} \ln 5$$

$$= \frac{\ln 5}{2} \cdot \frac{5^{\sqrt{z}}}{\sqrt{z}}$$

(92)

$$\int \frac{x \cdot 2x^2}{1 + x^2} dx$$

$$u = 1 + 2x^2$$

↓

$$2x$$

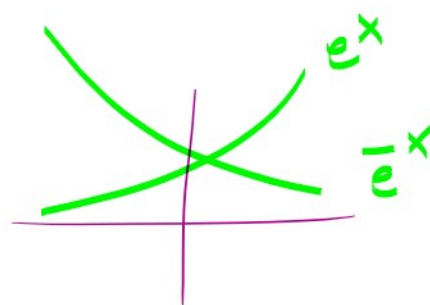
$$\int \frac{\frac{du}{2 \ln 2}}{u}$$

$$du = 0 + 2^x (2x) \ln 2$$

$$du = 2x 2^x \ln 2 dx$$

$$\frac{du}{2 \ln 2} = x 2^x dx$$

$$\frac{1}{\ln 4} \int \frac{du}{u}$$

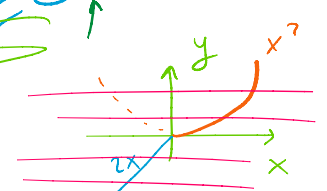


$$\frac{1}{\ln 4} \ln |u| + C$$

$$\frac{1}{\ln 4} \ln \left(1 + 2^{x^2} \right) + C$$

$$\frac{1}{\ln 4} \ln \left(1 + 2^{x^2} \right) + C$$

Exp 15 $f(x) = \begin{cases} 2x & , x < 0 \\ x^2 & , x \geq 0 \end{cases}$



1-1 ?? Yes

$$f(x_1) = f(x_2)$$

$x_1, x_2 \in D_1(f) = (-\infty, 0)$

$$2x_1 = 2x_2$$

$$x_1 = x_2$$

$$x_1, x_2 \in D_2(f) = [0, \infty)$$

$$f(x_1) = f(x_2)$$

$$x_1^2 = x_2^2$$

$$\sqrt{x_1^2} = \sqrt{x_2^2}$$

$$|x_1| = |x_2|$$

$$x_1 = x_2$$

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