FUNDAMENTALS OF MECHANICS

Chapter 1

Structural Requirements

To perform its function of supporting a building in response to whatever loads may be applied to it, a structural element/ system must possess four properties:

- it must be capable of achieving a state of equilibrium,
- it must have adequate strength and integrity,
- it must have adequate rigidity, and
- it must be stable.

1- Equilibrium

- A structure is said to be in static equilibrium if the resultant of the external forces acting on the body - including the supporting forces called reactions -is zero.

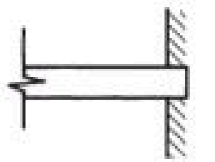
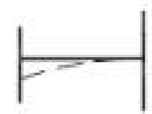
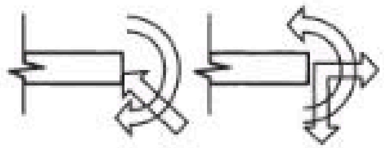
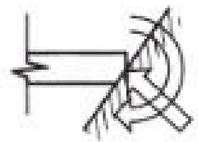
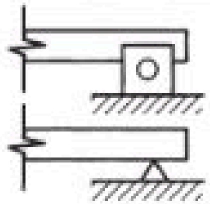
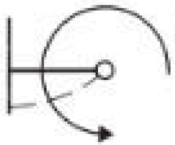
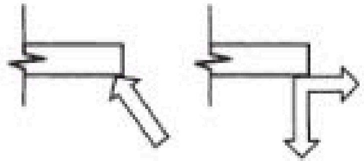
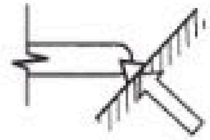
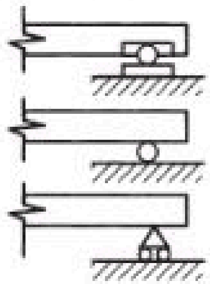
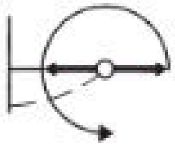
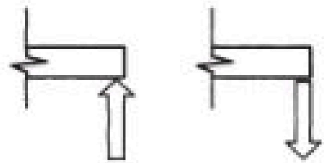
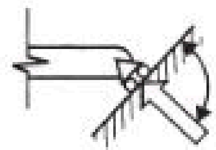
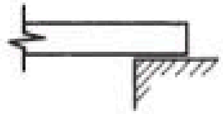
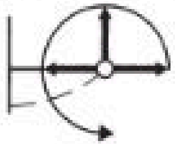
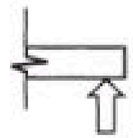
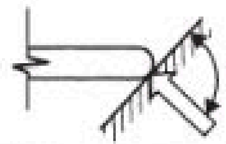
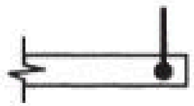
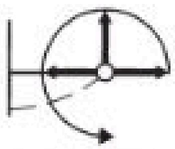
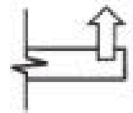
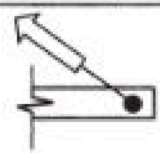
$$\sum \boldsymbol{Forces} = \mathbf{0}$$

2D	3D	
$\sum F_x = 0$	$\sum F_x = 0$	$\sum M_x = 0$
$\sum F_y = 0$	$\sum F_y = 0$	$\sum M_y = 0$
$\sum M_z = 0$	$\sum F_z = 0$	$\sum M_z = 0$

Types of Supports

- Supports can be classified into several basic categories based on their dominant behavior. These categories are hinges/pins, rollers/rockers, fixed, or links.
- Supports in reality never provide perfect restraint against translation or rotation. However, in most cases, each support has a dominant behavior that can be adequately captured by one of the idealized supports.

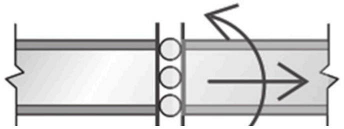
Types of Supports

Type of connection	Typical symbols	Types of translations and rotations the connection allows	Type of forces that can be developed at the connection	Types of forces that can be developed when the support is inclined
Fixed support				
Pinned support				
Roller support				
Simple support				
Cable support				

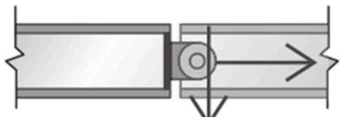
Connections (Joints)

- Connections between members in planar structures by default transfer two orthogonal forces (i.e., axial and shear) and moments about the axis perpendicular to the plane.

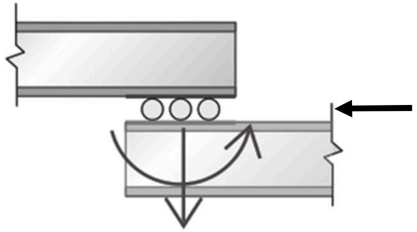
- Some connections are specifically designed to not transfer one of these internal forces between adjacent members. In other words, the connection releases one of these forces. There are three possible releases, namely a shear release/slider, a moment release/internal hinge, and an axial release.



shear release/slider



moment release/
internal hinge



Typically this type of connection does not transfer moment

axial release

Stability and Statical Determinacy

- The conditions of determinacy, indeterminacy, and instability of beams and frames can be stated as follows:

$$3m + r < 3j + C \text{ Structure is statically unstable}$$
$$3m + r = 3j + C \text{ Structure is statically determinate}$$
$$3m + r > 3j + C \text{ Structure is statically indeterminate}$$

where

r = number of support reactions.

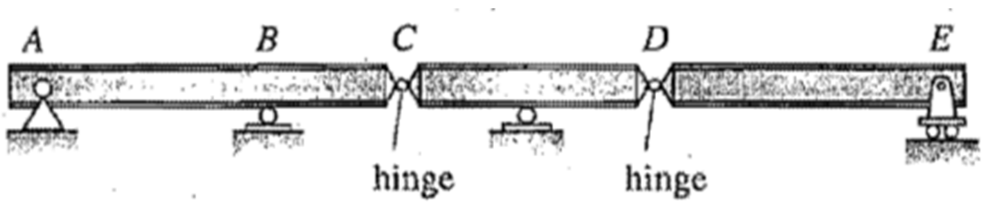
C = equations of condition (two equations for one internal roller and one equation for each internal pin).

m = number of members and j = number of joints.

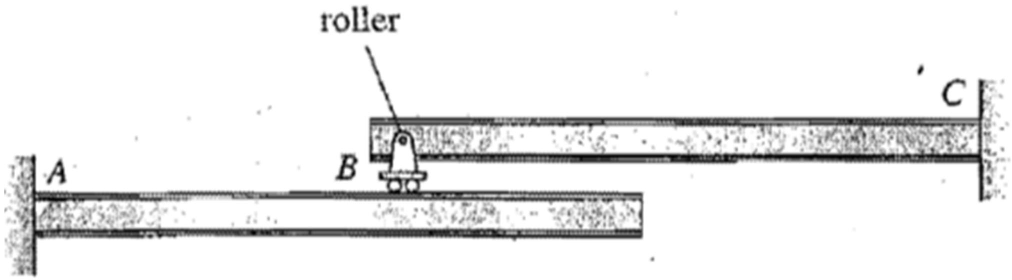
- **Stability and statical determinacy depend upon the structure's configuration; they are not dependent upon the loads applied to the structure.**

Statical Determinacy

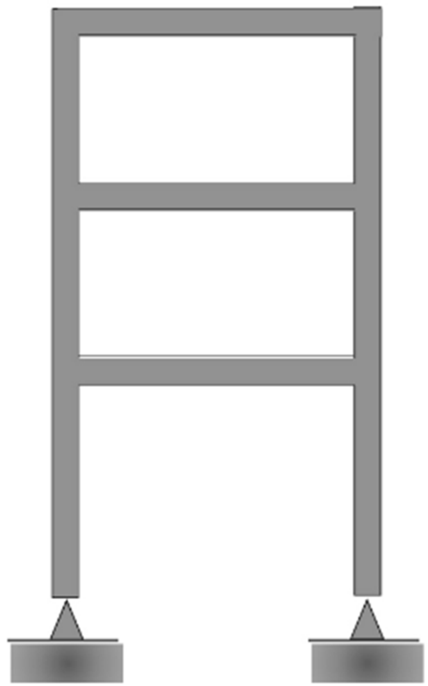
Example: determine the indeterminacy of the following



r=5, m=5, j=6, C=2 -> stable and determinate



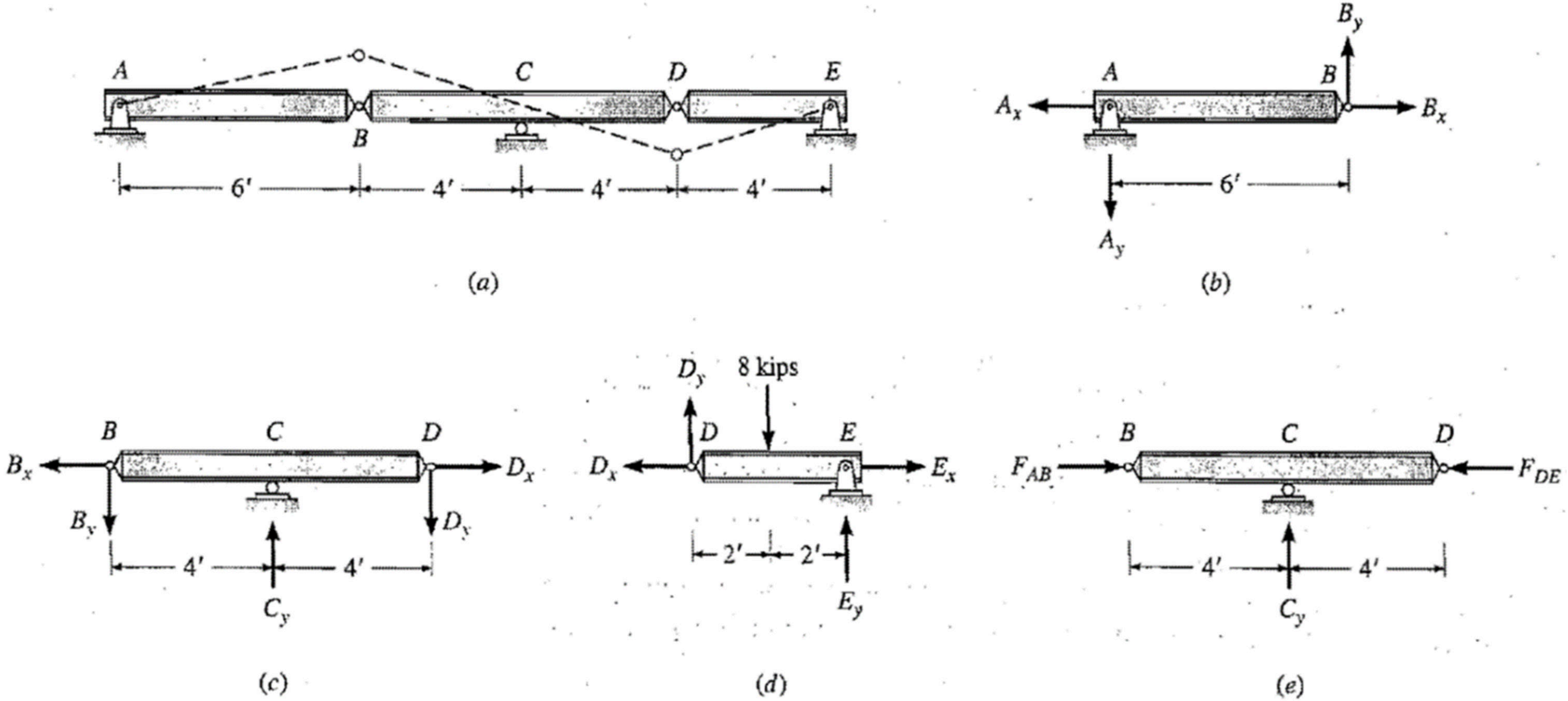
r=6, m=2, j=3, C=2 indeterminate to first degree



r=4, m=9, C=0, j=8. the frame is statically indeterminate to the 7 degrees.

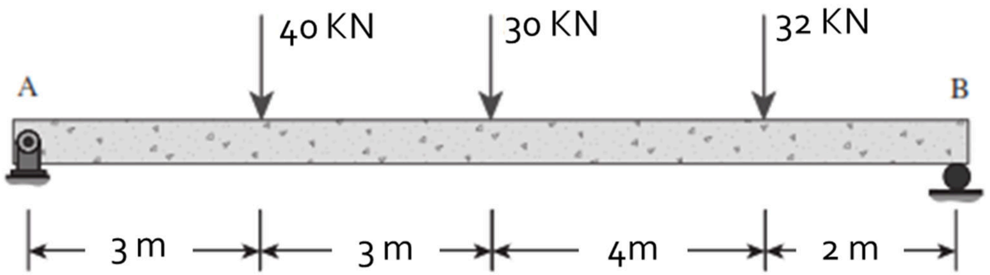
Determinacy and Stability

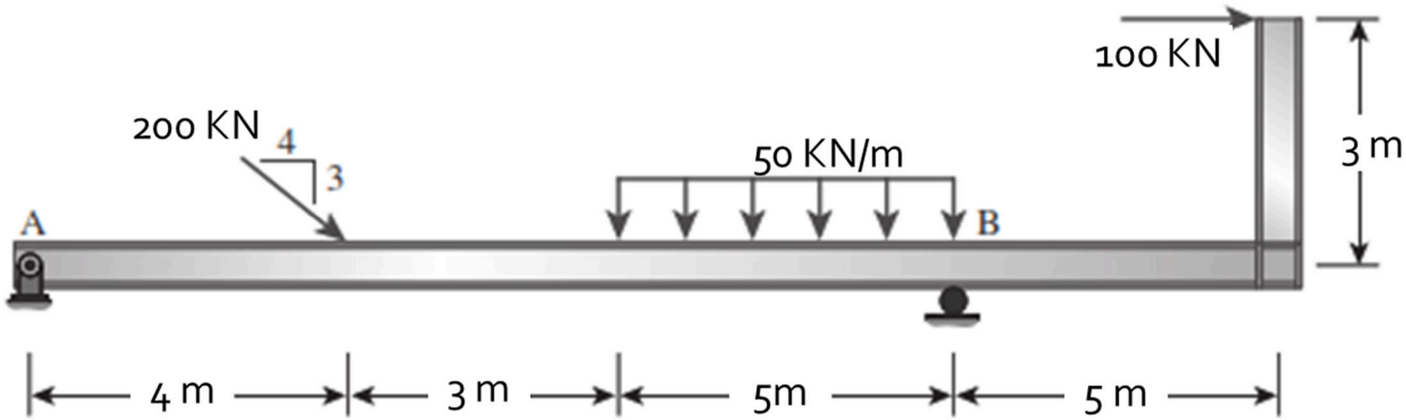
Example: determine the indeterminacy of the following beam. r=5, m=4, j=5, C=2 -> Determinate. However, the structure is unstable.

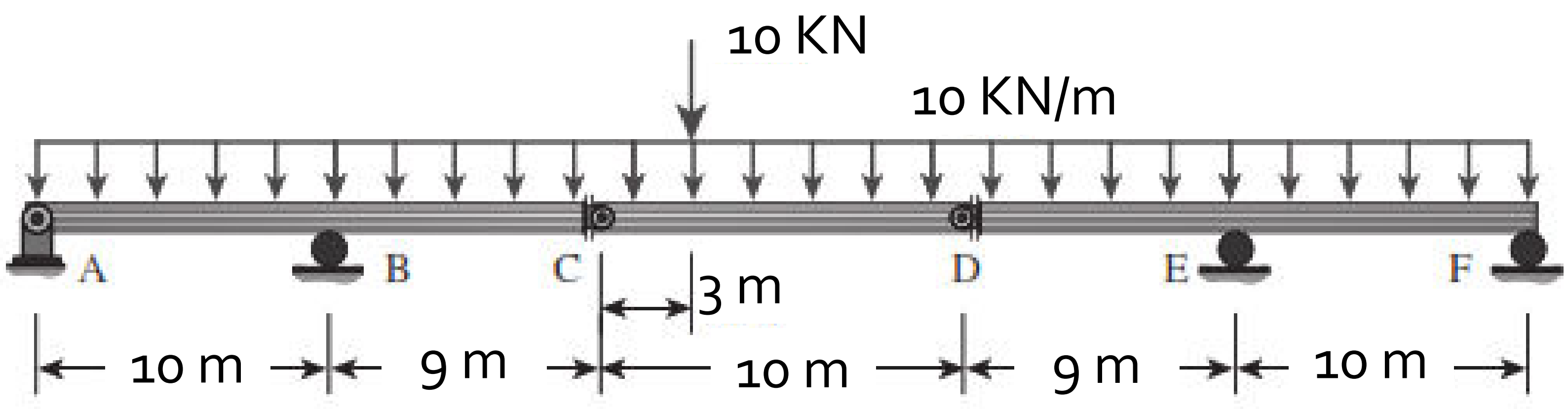


Reactions

- Example: determine the reactions of the following structures





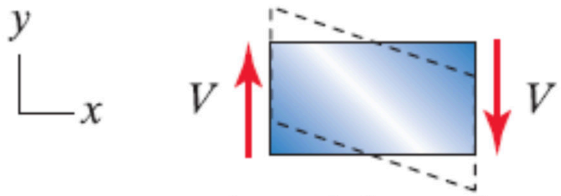


Internal Forces in Members

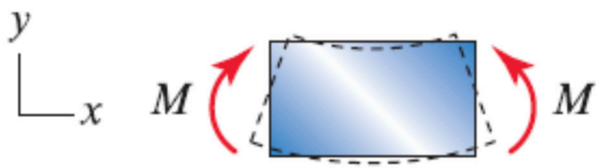
- Internal forces that develop on a particular cross-section of a structural member in two dimensions are :

1. The normal force or axial force (N), that gives rise to the axial deformation.
2. The shear force (V) that gives rise to the shear deformation.
3. The bending moment (M) that gives rise to the bending deformation.

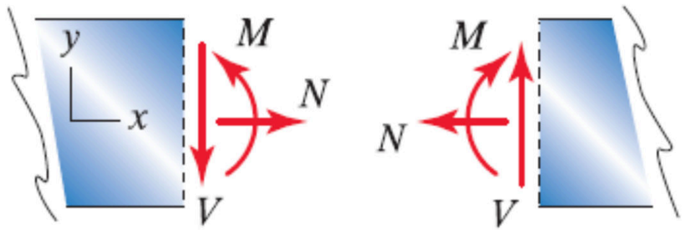
axial deformation



shear deformation



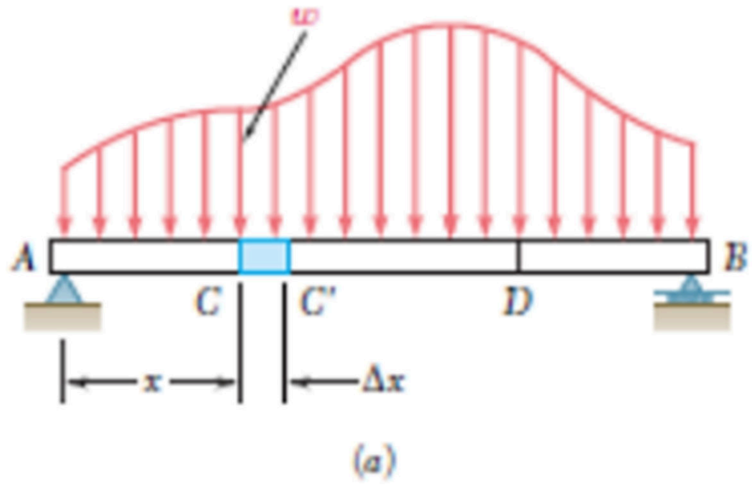
bending deformation



- **Sign Convention.** We will usually follow the sign convention shown in the figures to indicate the positive internal forces.

Relations of Load, Shear, And Bending Moment

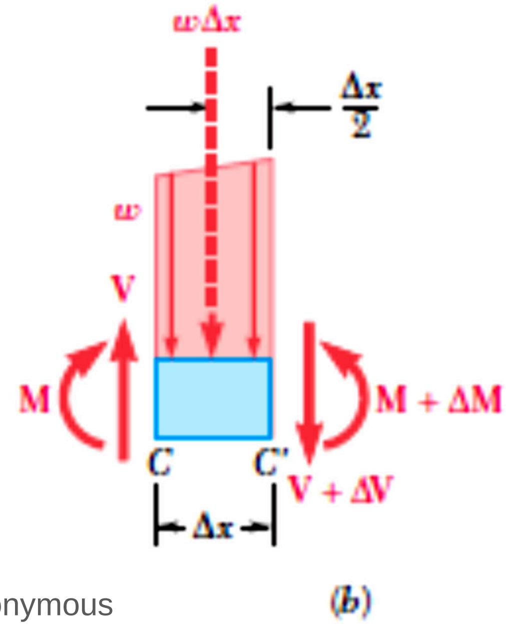
- <u>Relations between load and shear</u>



$$V - (V + \Delta V) - w\Delta x = 0$$

$$\frac{dV}{dx} = \lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = -w$$

$$V_D - V_C = -\int_{x_C}^{x_D} w dx = -(\text{area under load curve})$$

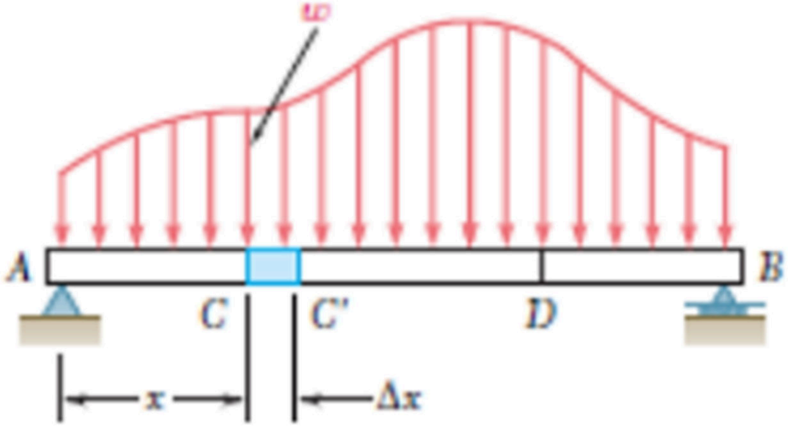


The slope of the shear diagram is equal to the distributed force's value.

Relations of Load, Shear, And Bending Moment

- <u>Relations between shear and bending moment</u>

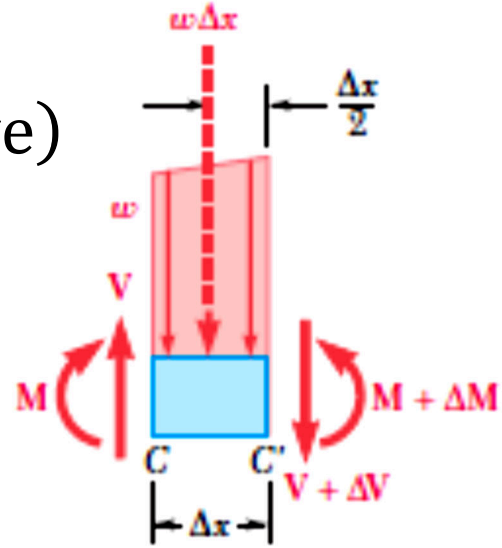
$$(M+\Delta M)-M-V\Delta x+w\Delta x\frac{\Delta x}{2}=0$$



(a)

$$\frac{dM}{dx}=\lim_{\Delta x\to 0}\frac{\Delta M}{\Delta x}=\lim_{\Delta x\to 0}\left(V-\frac{1}{2}w\Delta x\right)=V$$

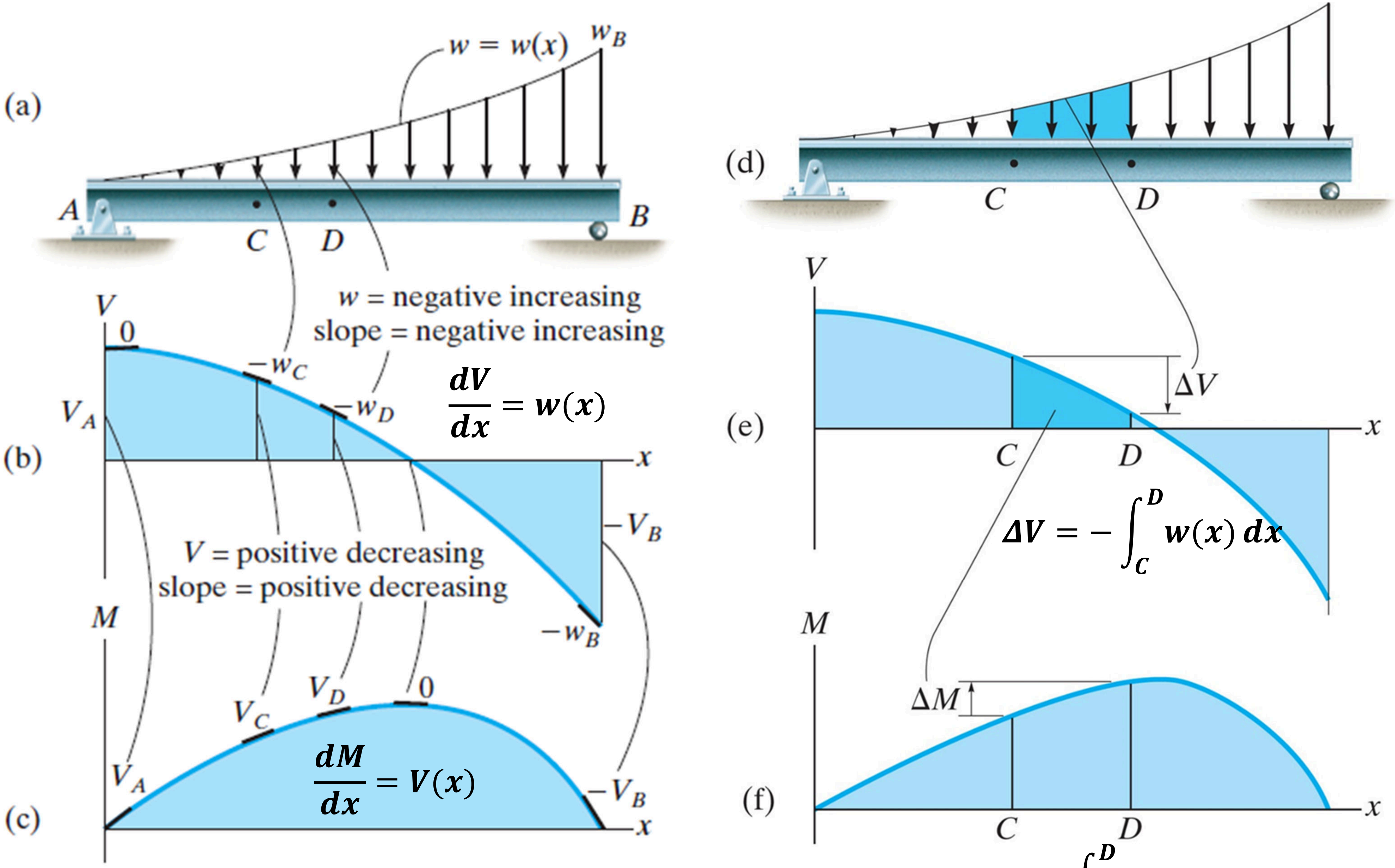
$$M_D-M_C=\int_{x_C}^{x_D}V\,dx=(\text{area under shear curve})$$



(b)

The slope of the moment diagram is equal to the value of the shear.

Drawing SF and BM diagrams



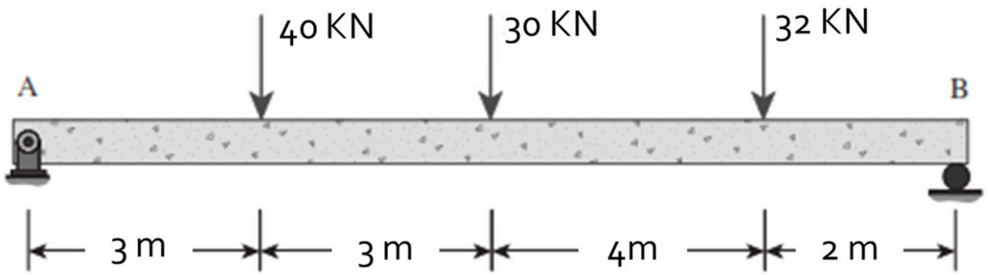
$$\frac{dV}{dx} = w(x)$$

$$\frac{dM}{dx} = V(x)$$

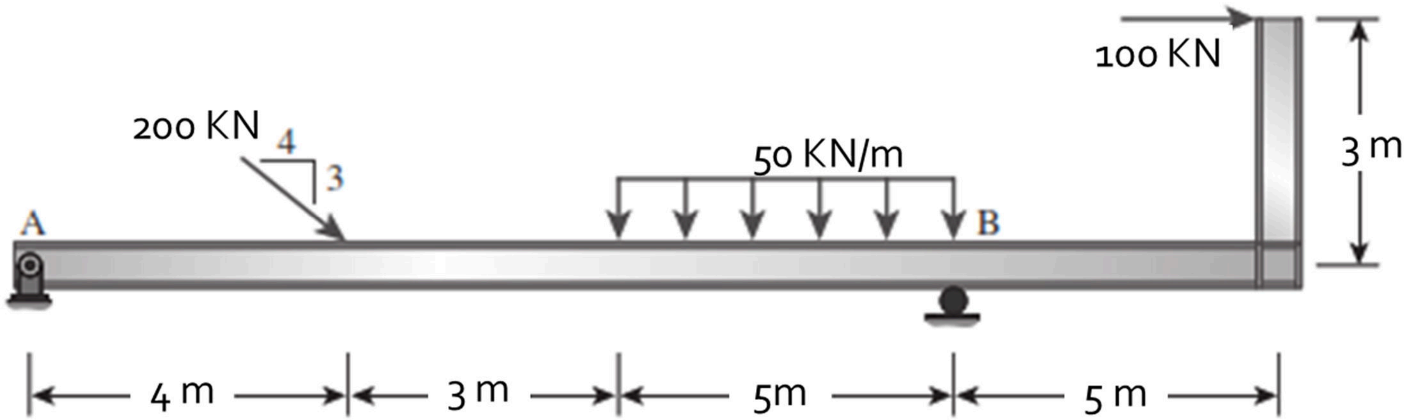
$$\Delta V = -\int_C^D w(x)\,dx$$

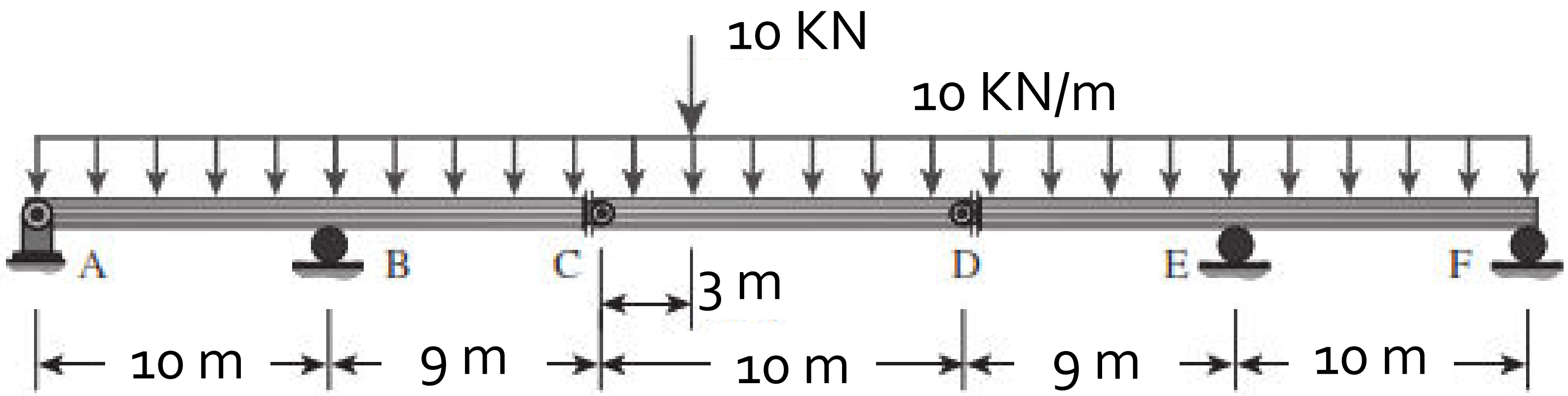
$$\Delta M = \int_C^D V(x)\,dx$$

Internal forces



- Example: Draw the SF & BM diagrams for the following structures



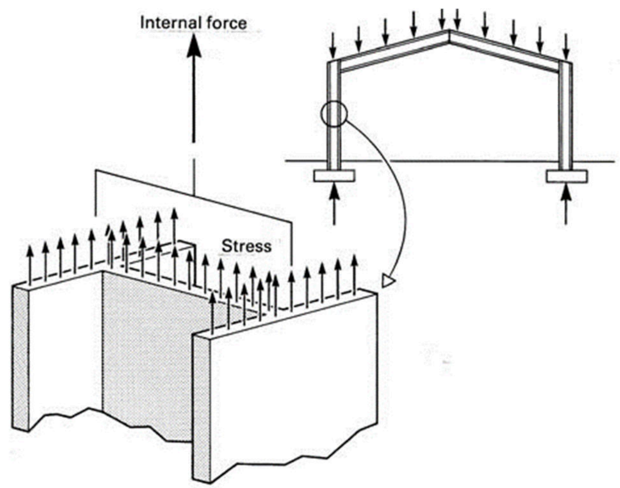


2- Strength

- The requirement for adequate strength is satisfied by ensuring that the stress levels that occur in a structure's various elements, when the peak loads are applied, are within acceptable limits.

$$\phi\sigma_u > \sigma_A$$

Where:
Φ is the safety factor
σ_u is material strength
σ_A is stress due to loads



- This is chiefly a matter of providing elements with cross-sections of adequate size, given the strength of the constituent material.
- Each material has its unique mechanical properties, including strength. In some materials, strength varies for different types of internal forces.

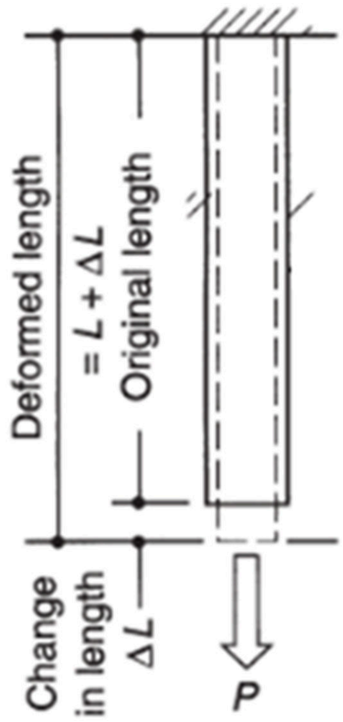
Axial stress and strain

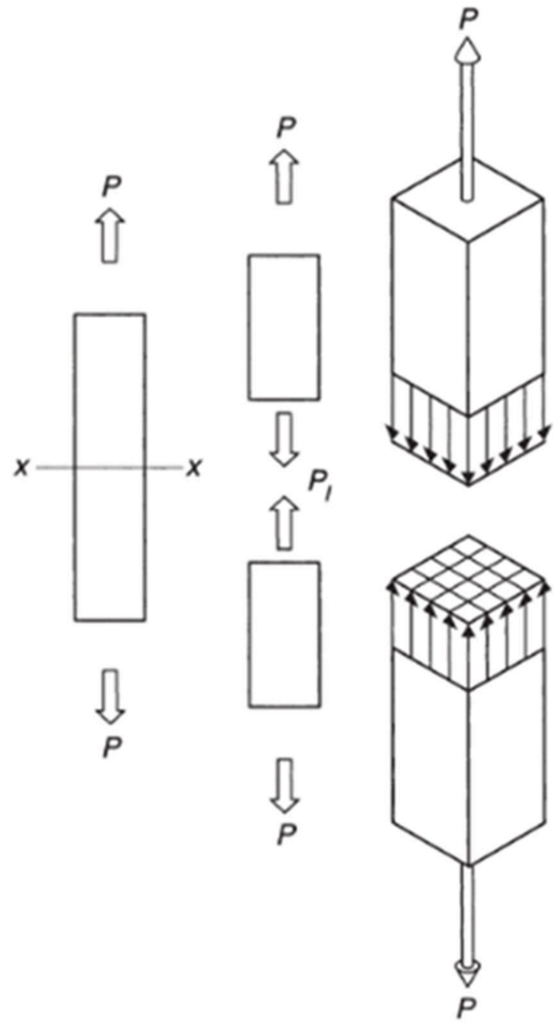
- For a Constant Load and Cross-Sectional Area:

- Axial Stress: $\sigma = \frac{P}{A}$; where A is the cross-sectional area
- Axial Strain: $\epsilon = \frac{\Delta L}{L}$
- Hook's law $\sigma = E\epsilon$

$$\rightarrow \Delta L = \frac{PL}{AE}$$

E: Modulus of elasticity

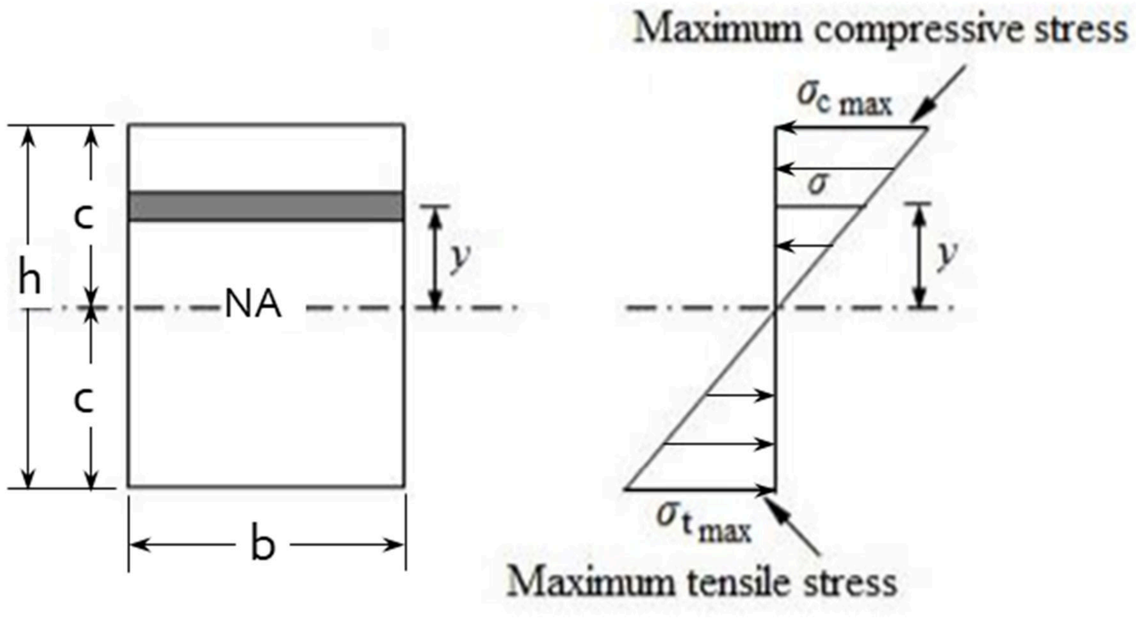




- To design axially loaded member

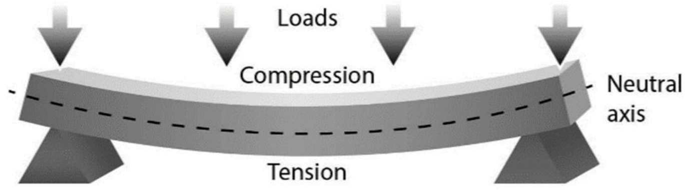
$$A_{Required} = \frac{Force}{Allowable\ stress} = \frac{P}{\sigma_{all}}$$

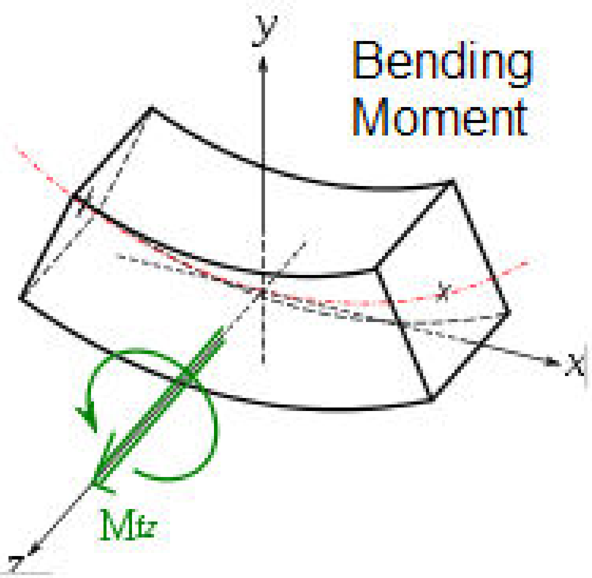
Bending stress and strain in Beams

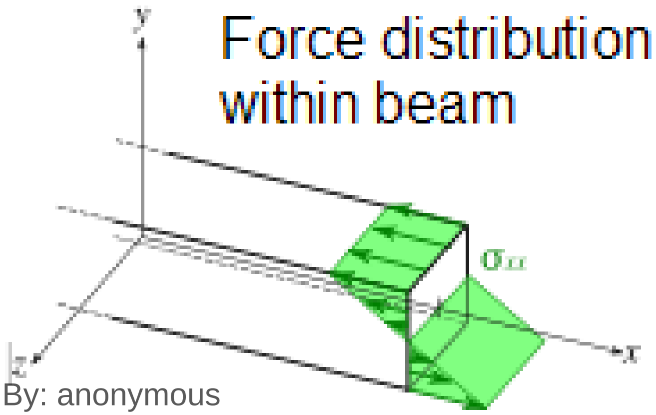


(a) Cross section

(b) Stress distribution diagram







- **Flexure Formula**

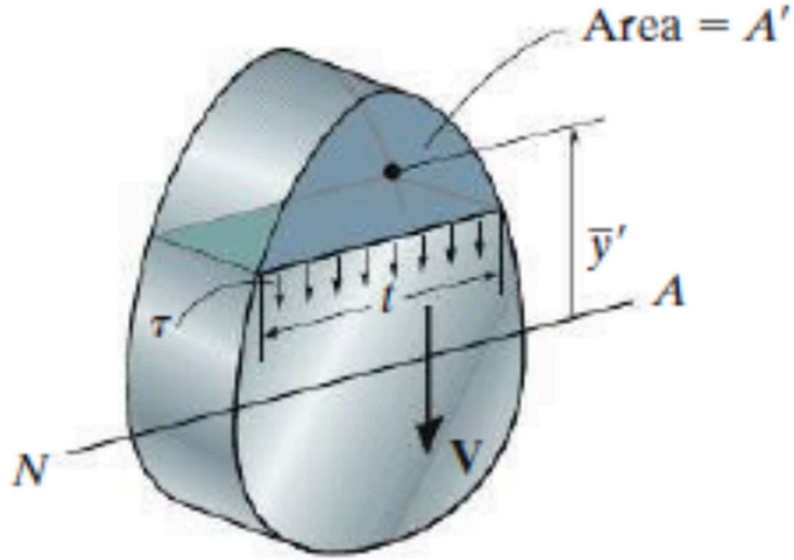
$$\sigma_y = \frac{My}{I}; \quad \sigma_{max} = \frac{Mc}{I}$$

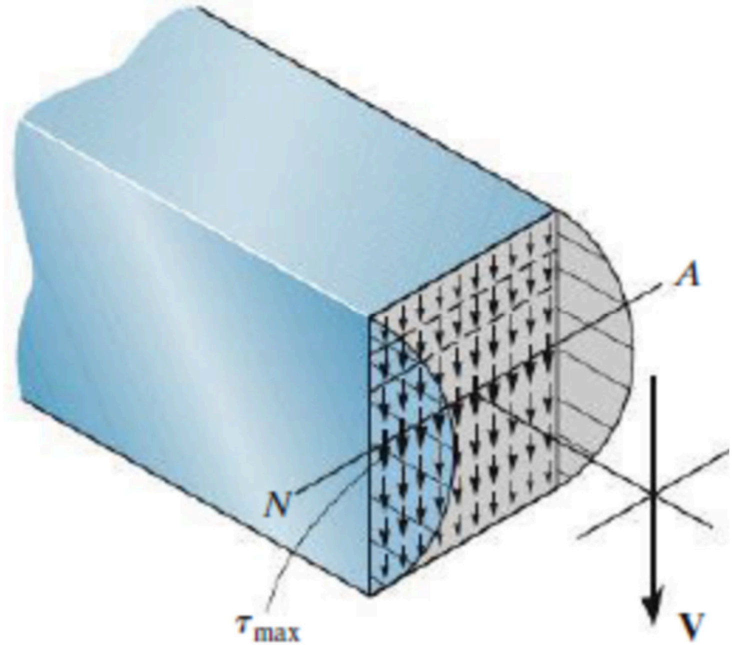
I: the section's second moment of area

Sheer stress

$$\tau = \frac{VQ}{It}$$

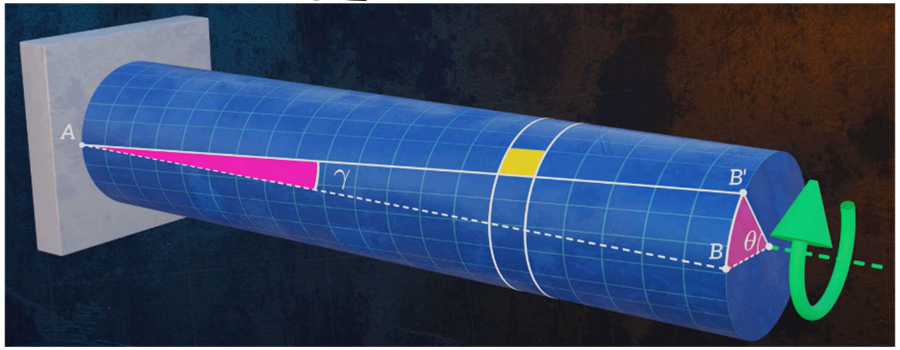
- **t** = the shear stress in the member at the point located at a distance y' from the neutral axis.
- V = the internal resultant shear force.
- I = the moment of inertia of the entire cross-sectional area
- t = the width of the member's cross-sectional area, measured at the point where t is to be determined
- $Q = \bar{y}'A'$





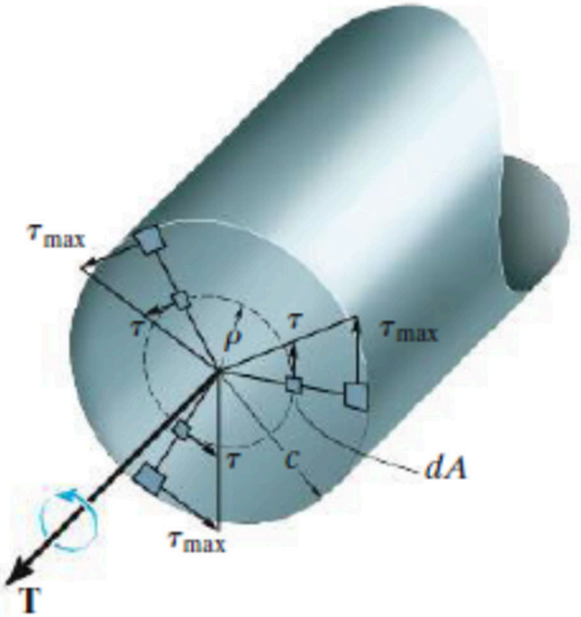
Torsional stress and strain

$$\tau = \frac{T\rho}{J}$$



<u>Angle of Twist</u> for Constant Torque and Cross-Sectional Area is determined from

$$\theta = \frac{TL}{JG}$$



t = the shear stress in the member at the point located at a distance ρ from the center.

T = The applied torque.

J = Polar moment of inertia

G = shear modulus of the material

3- Serviceability - Rigidity

- The serviceability of the building is its fitness for use that extends beyond strength considerations. The structure could be strong enough to carry the required loads but the building function still be impaired if the deflection limits are exceeded.

- Building codes usually propose certain deflection limits such as those shown in the table below.

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\ell/180^*$
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\ell/360$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)†	$\ell/480^‡$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\ell/240^§$

Serviceability - Rigidity

To insure the fulfilment of the serviceability requirement the determination of the member sizes is carried out either by using geometric rules (such as minimum ratios of span to depth for beams as shown in the table) or by deflection calculations

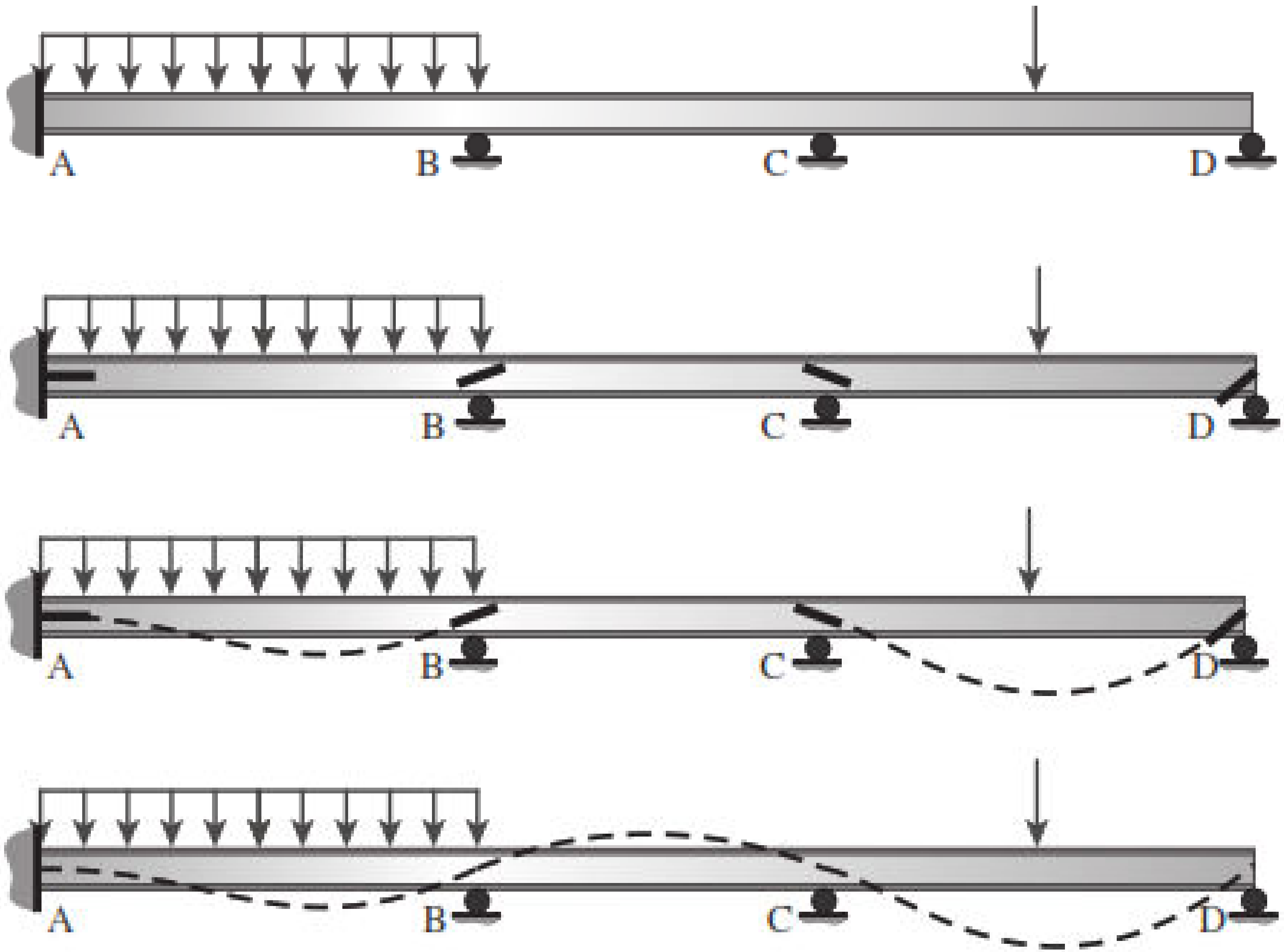
Minimum Thickness of Beams and One-way Slabs

Restraint	**simply supported**	**one end continuous**	**both ends continuous**	**cantilever**
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections			
Solid one-way slabs	$l/20$	$l/24$	$l/28$	$l/10$
Beams or ribbed one-way slabs	$l/16$	$l/18.5$	$l/21$	$l/8$

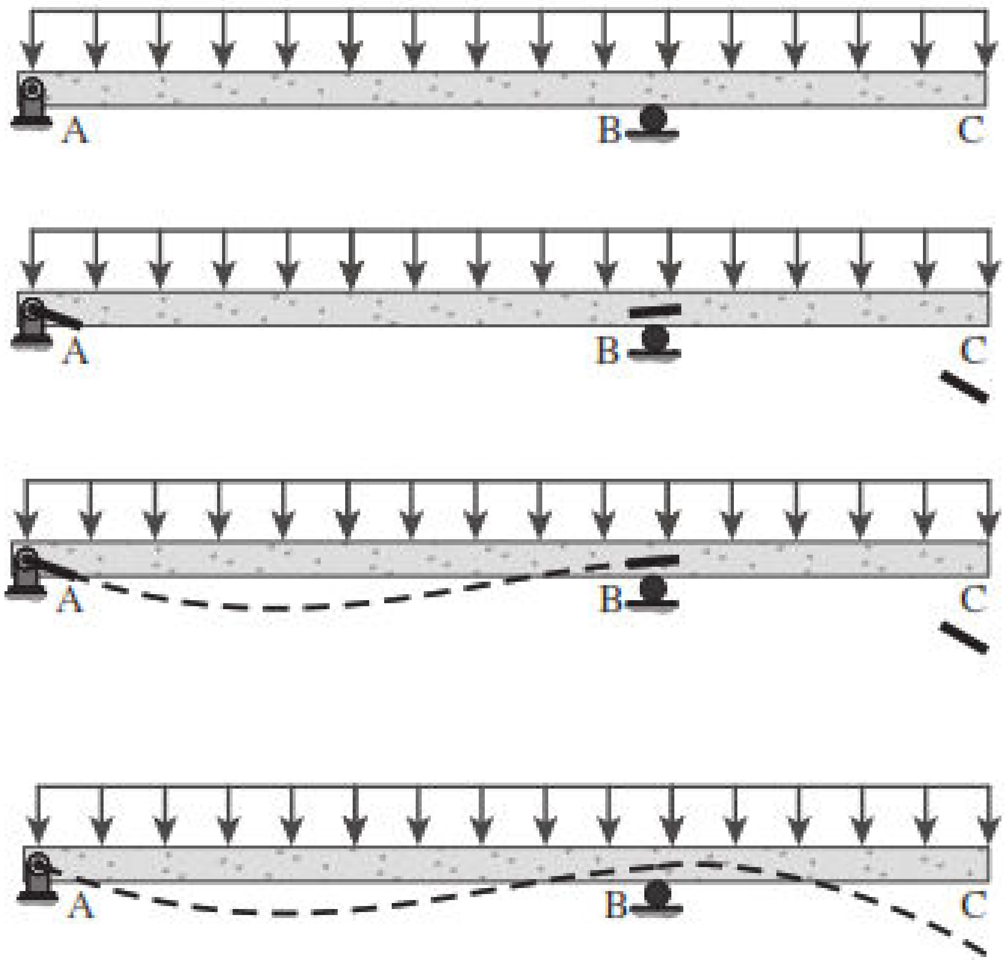
Deflection

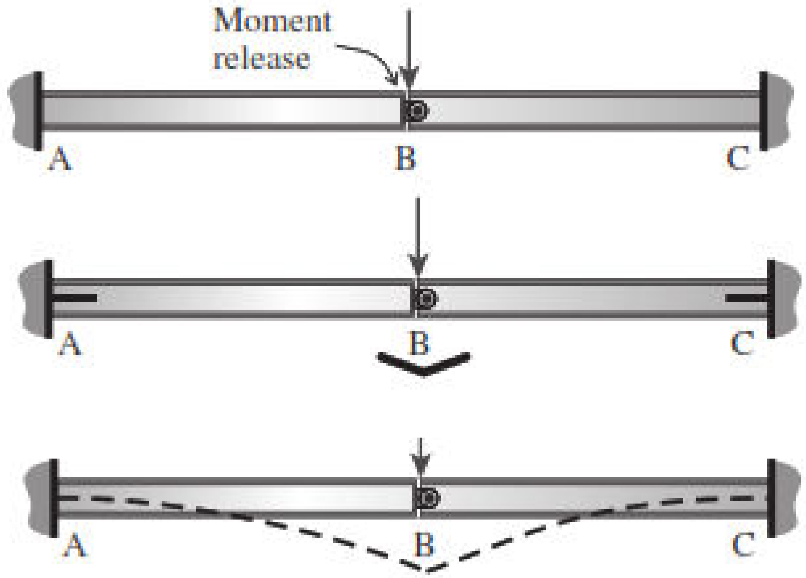
- Understanding the displacement behavior of structural systems is a very important part of understanding how structures perform.
- The engineer should be able to sketch the anticipated deformed shape of structures under load before making actual calculations.
- Such a practice provides an appreciation of the behavior of the structure and provides a qualitative check of the magnitudes and directions of the computed displacements.

Sketching Deformed Shapes of Structures



Sketching Deformed Shapes of Structures





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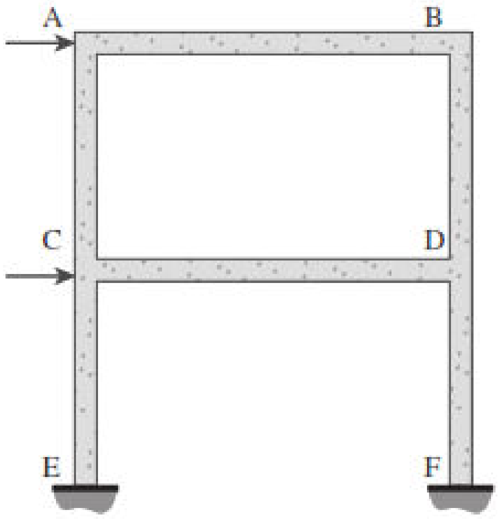
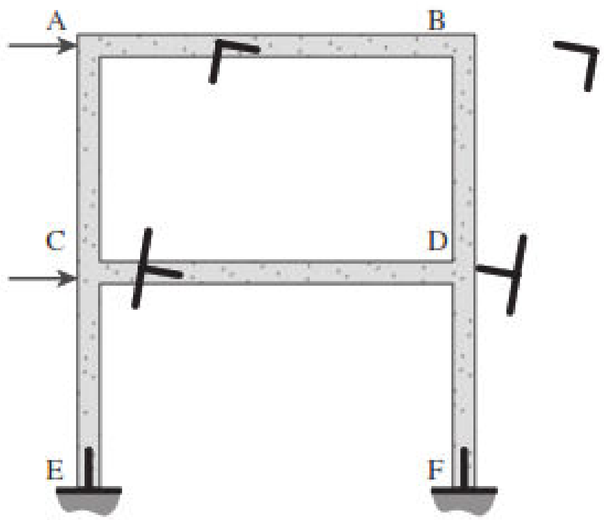
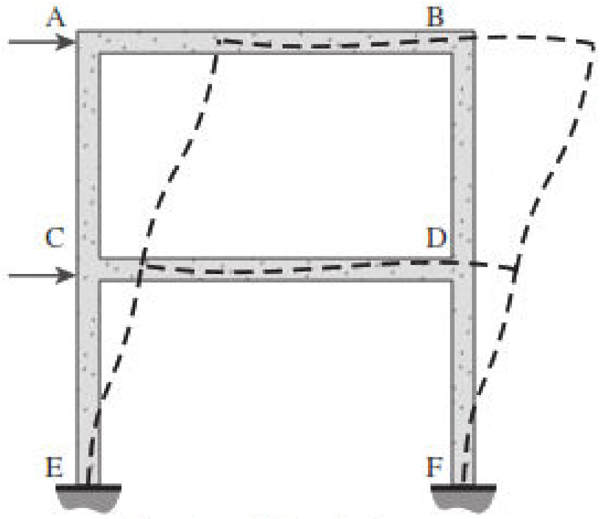




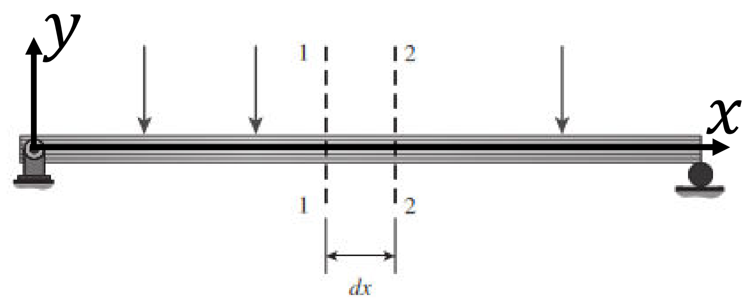
Joint displacement



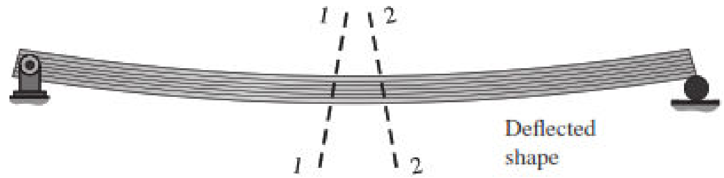
Complete deflected shape

Elastic Beam Theory

$$\varepsilon = \frac{ds' - ds}{ds} = \frac{(R+c)d\theta - Rd\theta}{Rd\theta} = \frac{c}{R}$$

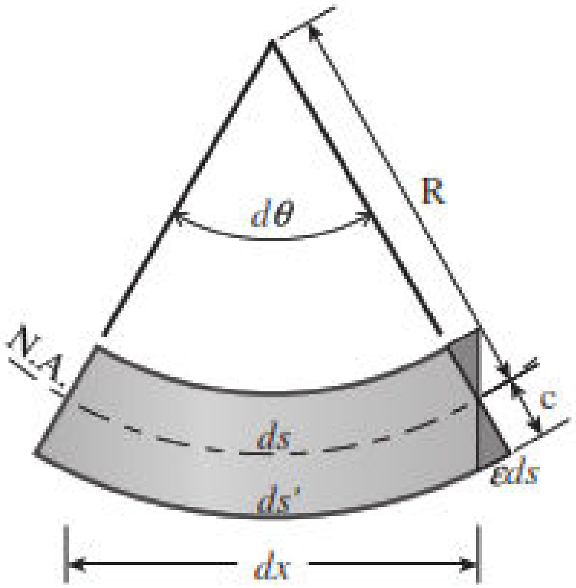


$$\varepsilon = \frac{\sigma}{E}; and\ \sigma = \frac{Mc}{I}$$



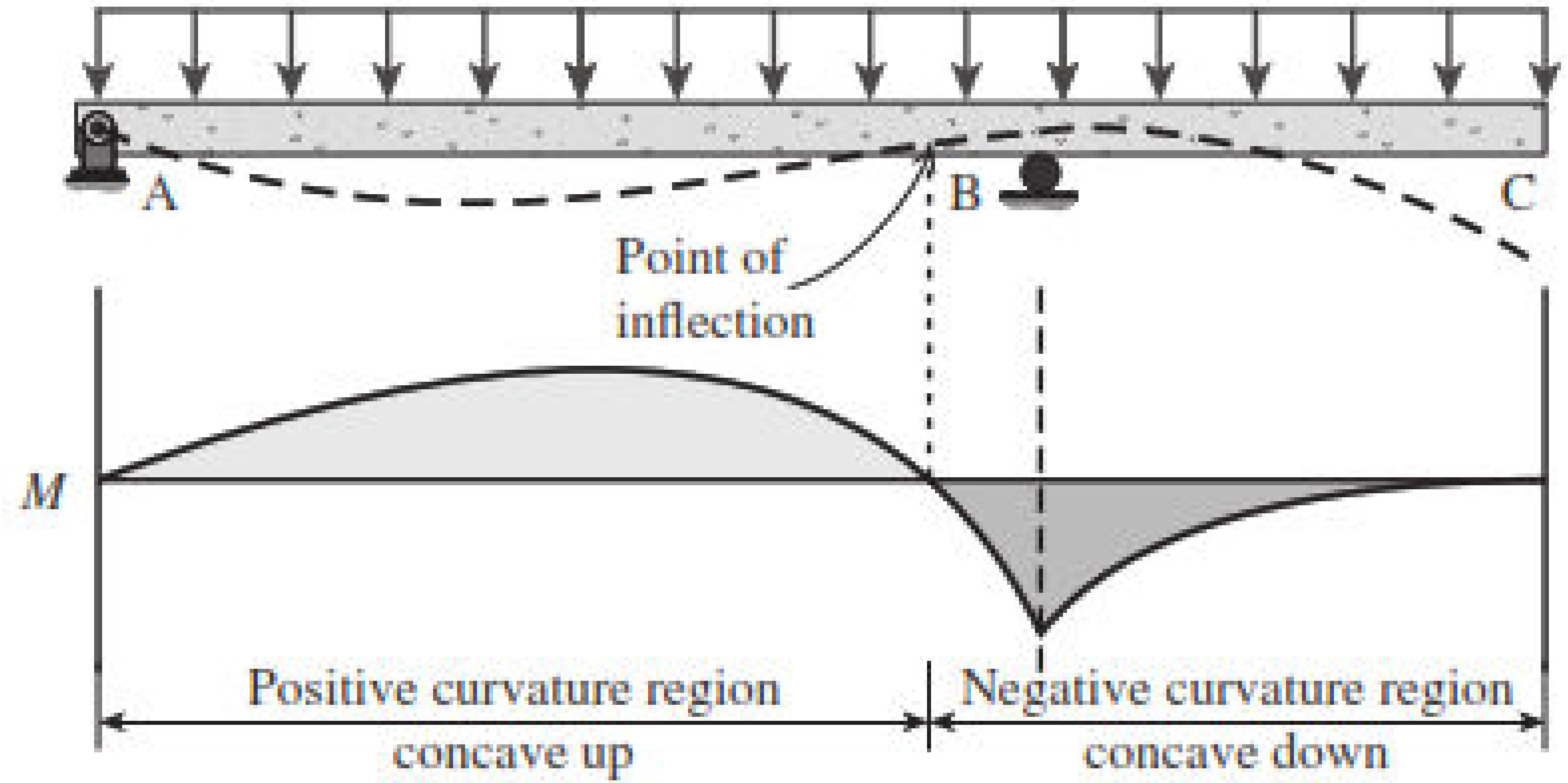
$$\rightarrow\ \frac{1}{R} = \frac{M}{EI}$$

$$\frac{1}{R} = \frac{d^2y/dx^2}{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}} \approx \frac{d^2y}{dx^2}$$



$$\Rightarrow\ \boxed{\frac{d^2y}{dx^2} = \frac{M}{EI}}$$

Elastic Beam Theory



As the second derivative of the elastic curve equals M/EI, the moment diagram can be used to plot the deflected shape as shown above.

Deflection by Double Integration

- The equations for the slope (θ) and the deflection (y) as a function of x can be developed by solving the second-order differential equation given before. The solution process for this equation is straightforward since it can be solved by performing two successive integrations (i.e., double integration).

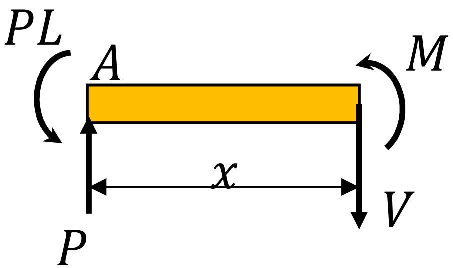
$$\frac{dy}{dx} = slope(\theta) = \int \frac{M}{EI} dx$$

$$y = \iint \frac{M}{EI} dx$$

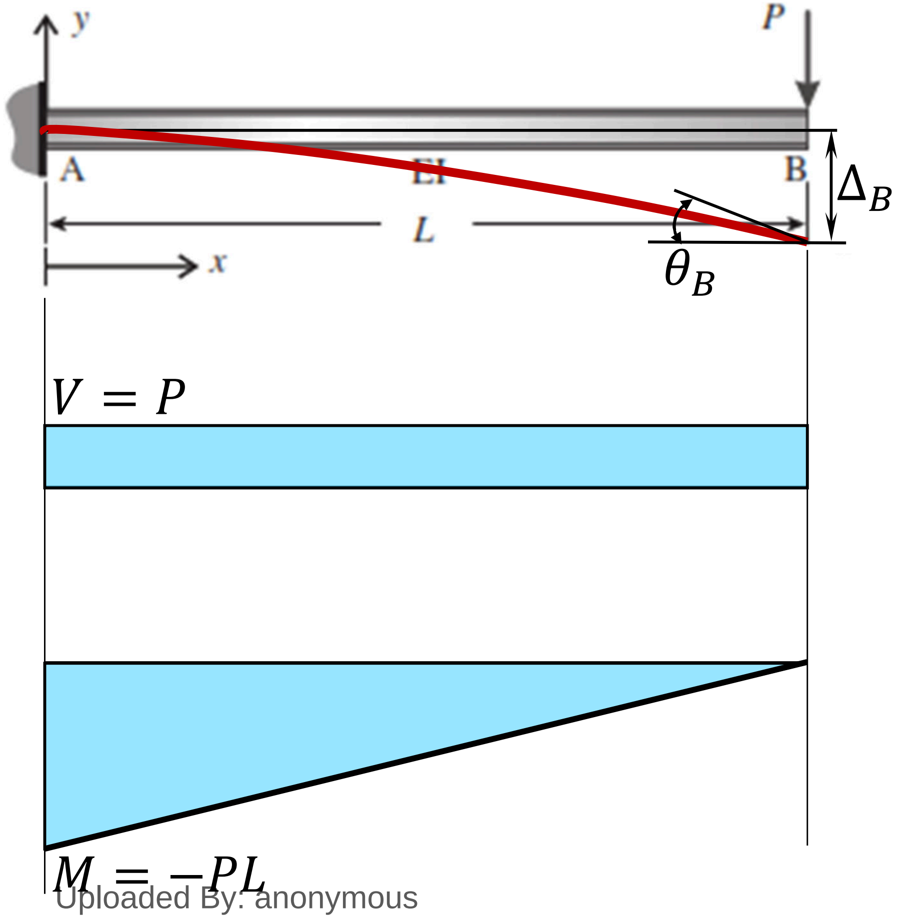
Double integration of the moment equation for each beam segment produces two integration constants per segment. This means that we need an equal number of boundary conditions to solve for these unknowns.

Deflection by Double Integration

- Example: A cantilever beam is shown with a point load being applied to its tip. Use the double integration method to find the equations for the slope and displacement of this beam. Identify what the displacement and the slope are at the tip of the beam (point B).



$$M(x) = Px - PL$$



$$\theta = \frac{1}{EI}\int (Px - PL)dx = \frac{1}{EI}\left(\frac{Px^2}{2} + PLx + C_1\right)$$

$$y = \frac{1}{EI}\int \left(\frac{Px^2}{2} + PLx + C_1\right)dx$$

$$y = \frac{1}{EI}\left(\frac{Px^3}{6} + \frac{PLx^2}{2} + C_1x + C_2\right)$$

Deflection by Double Integration

Evaluate BC: $\theta(x=0)=0$

$$\theta(x=0)=\frac{P}{EI}\left(\frac{1}{2}(0)^2-L(0)+C_1\right)=0$$

$$\therefore \underline{\underline{C_1=0}}$$

We now use this known value for C_1 in future calculations.

Evaluate BC: $y(x=0)=0$

$$y(x=0)=\frac{P}{EI}\left(\frac{(0)^3}{6}-\frac{L(0)^2}{2}+(0)(0)+C_2\right)=0$$

$$\therefore \underline{\underline{C_2=0}}$$

$$\theta(x)=\frac{P}{EI}\left(\frac{1}{2}x^2-Lx\right); \qquad 0\le x\le L$$

$$y(x)=\frac{P}{EI}\left(\frac{x^3}{6}-\frac{Lx^2}{2}\right); \qquad 0\le x\le L$$

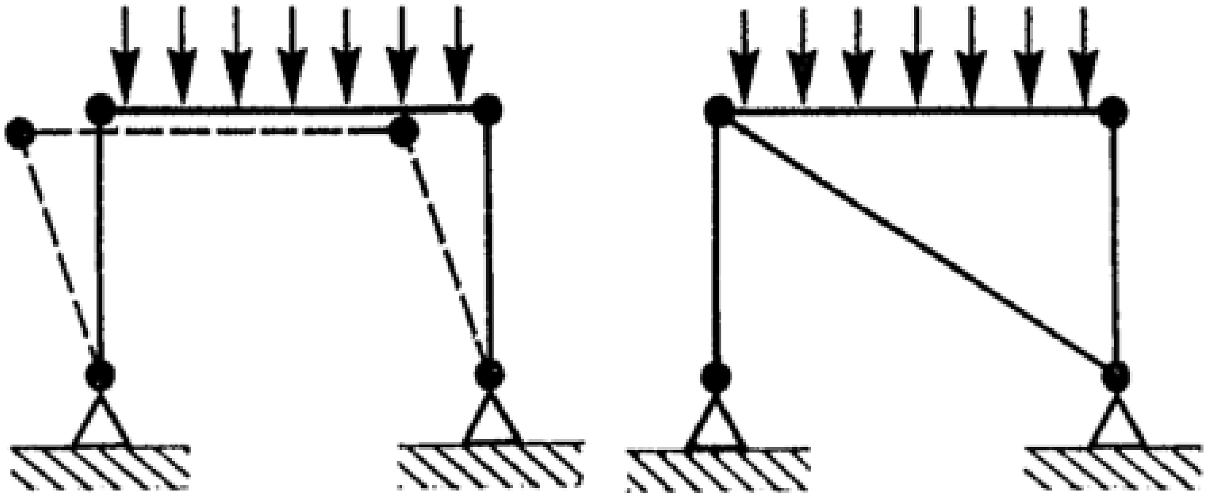
At x = L

$$\theta(L)=\frac{P}{EI}\left(\frac{1}{2}L^2-L(L)\right)=\underline{\underline{-\frac{PL^2}{2EI}}}$$

$$y(L)=\frac{P}{EI}\left(\frac{L^3}{6}-\frac{L(L)^2}{2}\right)=\underline{\underline{-\frac{PL^3}{3EI}}}$$

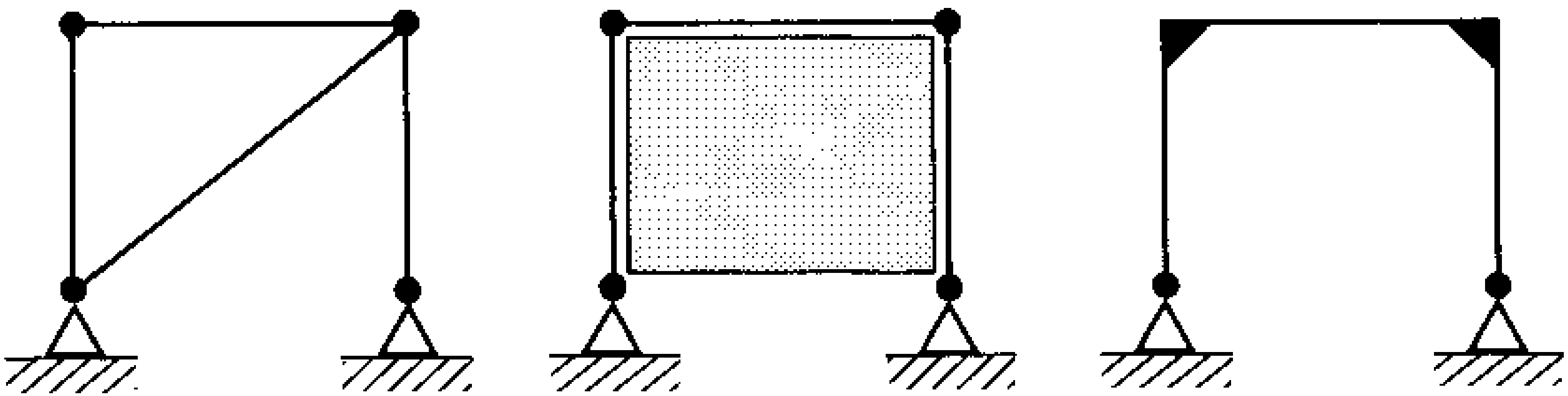
4- Geometric Stability

- Geometric stability is the property which preserves the geometry of a structure and allows its elements to act together to resist load.
- Stable systems revert to their original state following a slight disturbance whereas unstable systems progress to an entirely new state.



A rectangular frame with four hinges is capable of achieving a state of equilibrium but is unstable because any slight lateral disturbance to the columns will induce it to collapse. The frame on the right here is stabilized by the diagonal element which makes no direct contribution to the resistance of the gravitational load.

4- Geometric Stability



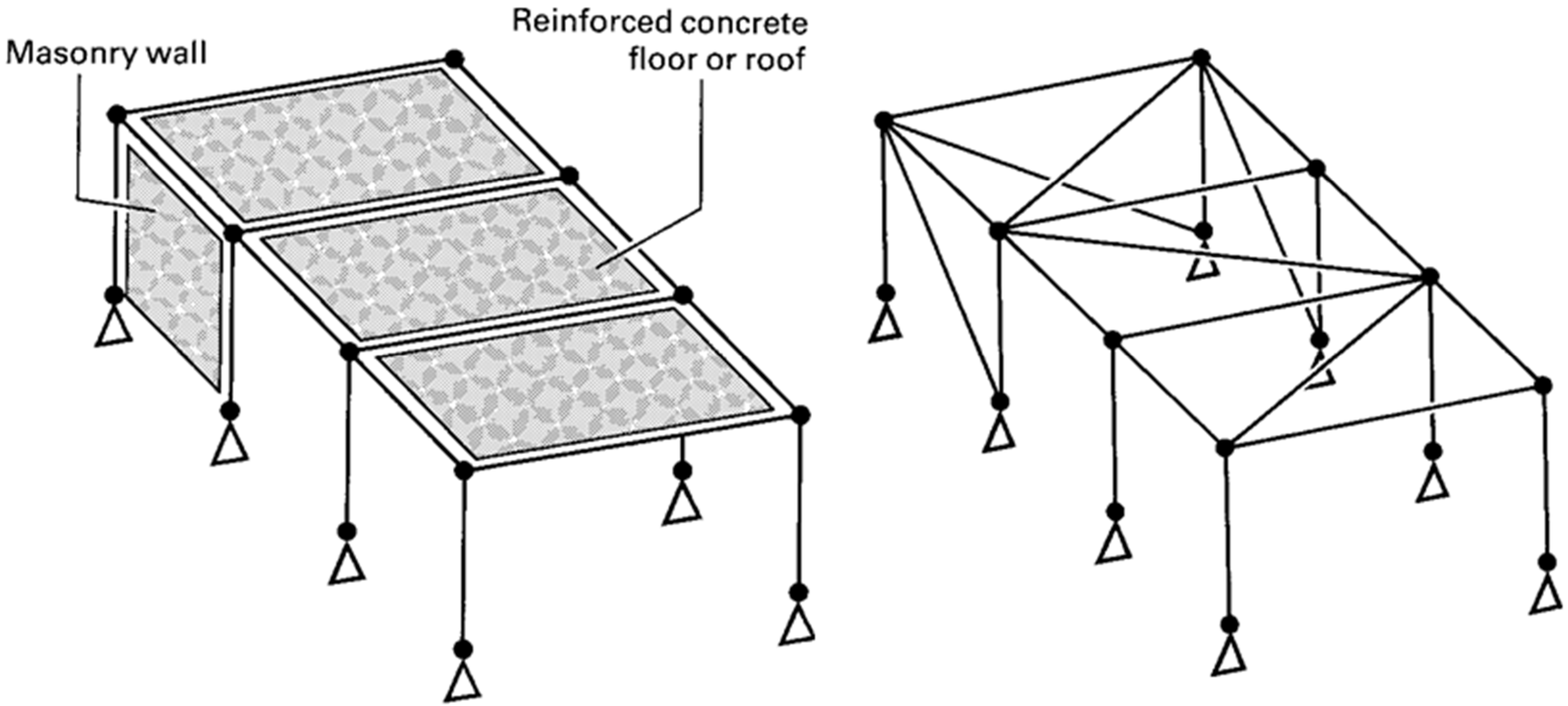
A rectangular frame can be stabilized by the insertion of

(a) a diagonal element or

(b) a rigid diaphragm, or

(c) by the provision of rigid joints. A single rigid joint is in fact sufficient to provide stability.

4- Geometric Stability

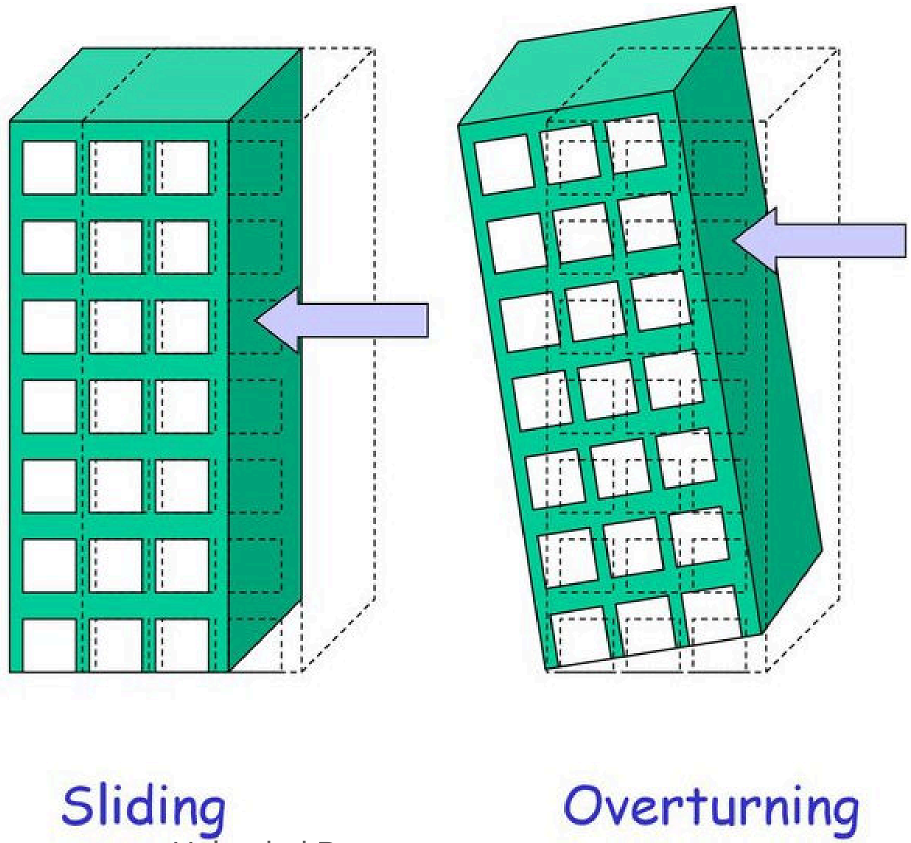


These frames contain the minimum number of braced panels required for stability.

4- Geometric Stability

Additional cases of instability

- Long slender structural member subjected to an axial compressive load can suddenly become unstable (buckle).
- Overall instability: structure overturning and/ or sliding

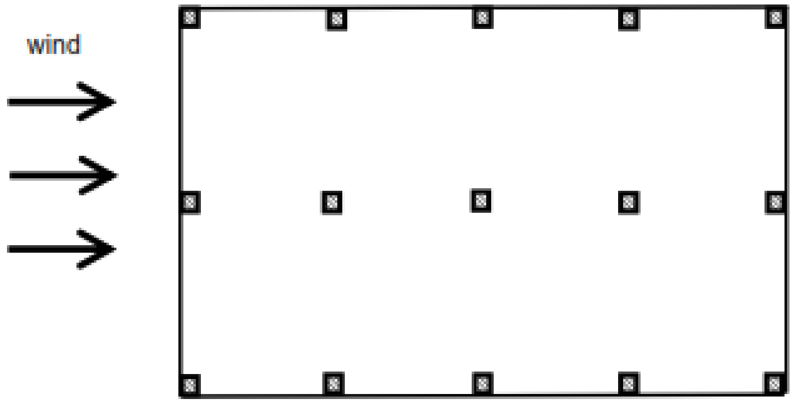


Stability of 'real' Structures

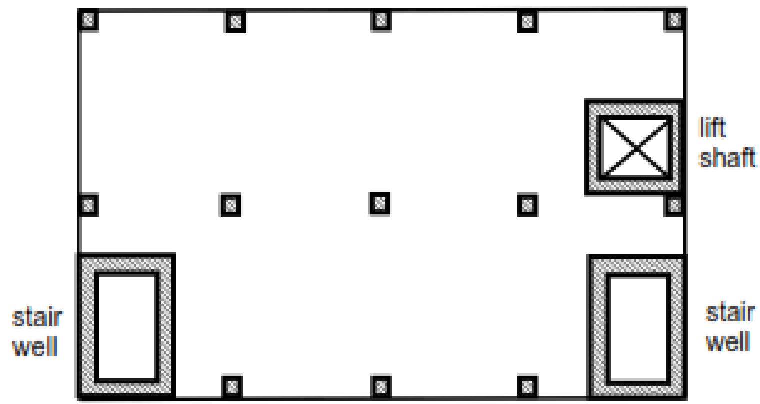
In practice, the stability of a structure is assured in one of three ways: Shear walls/stiff core; Cross-bracing; and Rigid joints.

Shear Walls.

- This form of stability is usually used in concrete buildings.
- Since most buildings have staircases and many have lift shafts, the walls that surround the staircases and lift shafts are often designed and constructed to perform this role.
- However, distribution of shear walls in the plan should be studied carefully to avoid irregular arrangement.



(a) Typical floor plan of reinforced concrete office building

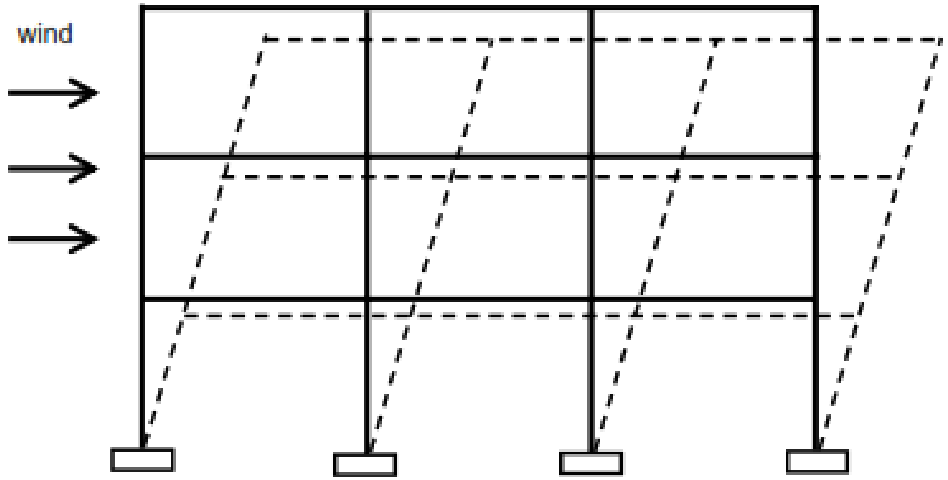


(b) Same floor plan with shear walls added

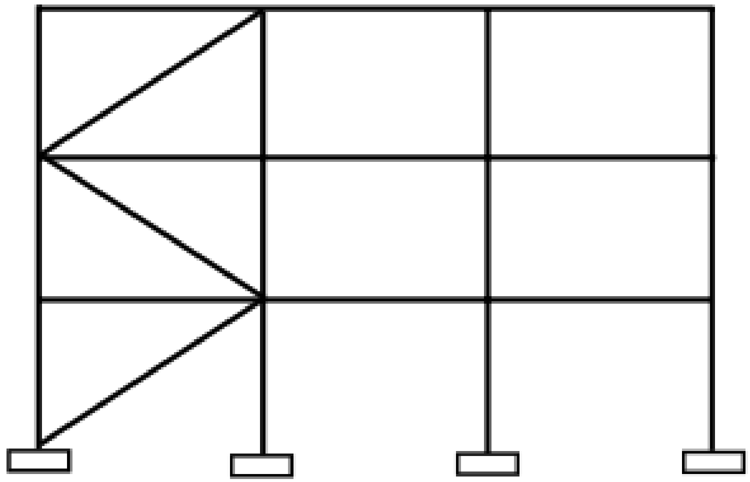
Stability of ‘real’ Structures

Cross-bracing

This form of stability is common in steel-framed buildings.



(a) Section through three-storey steel framed building

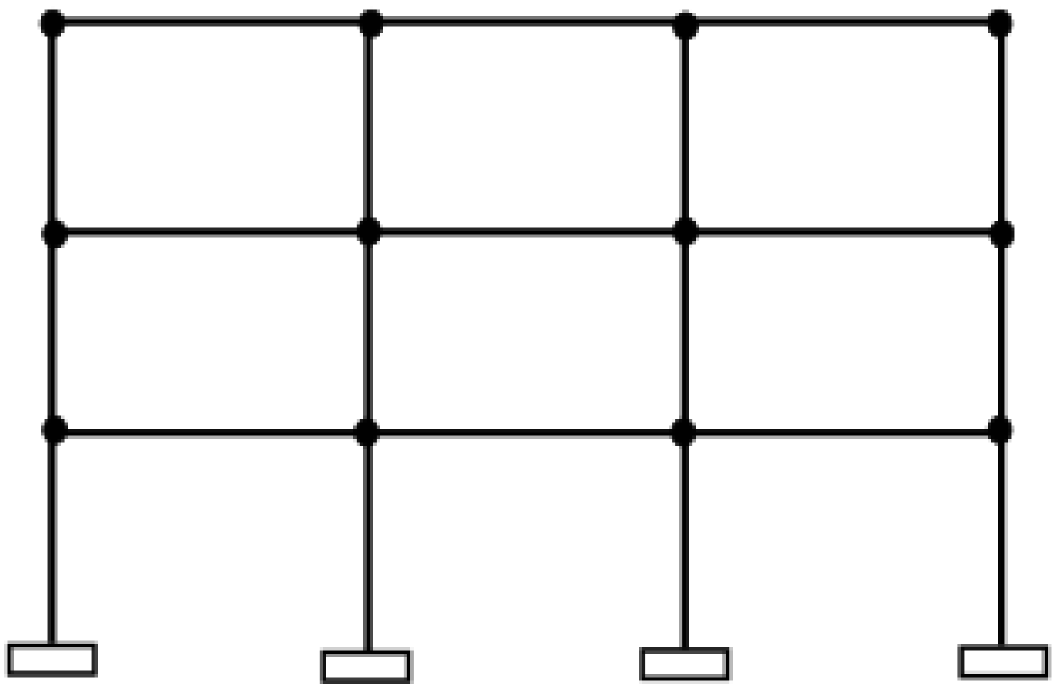


(b) Same section with diagonal bracing added

Stability of ‘real’ Structures

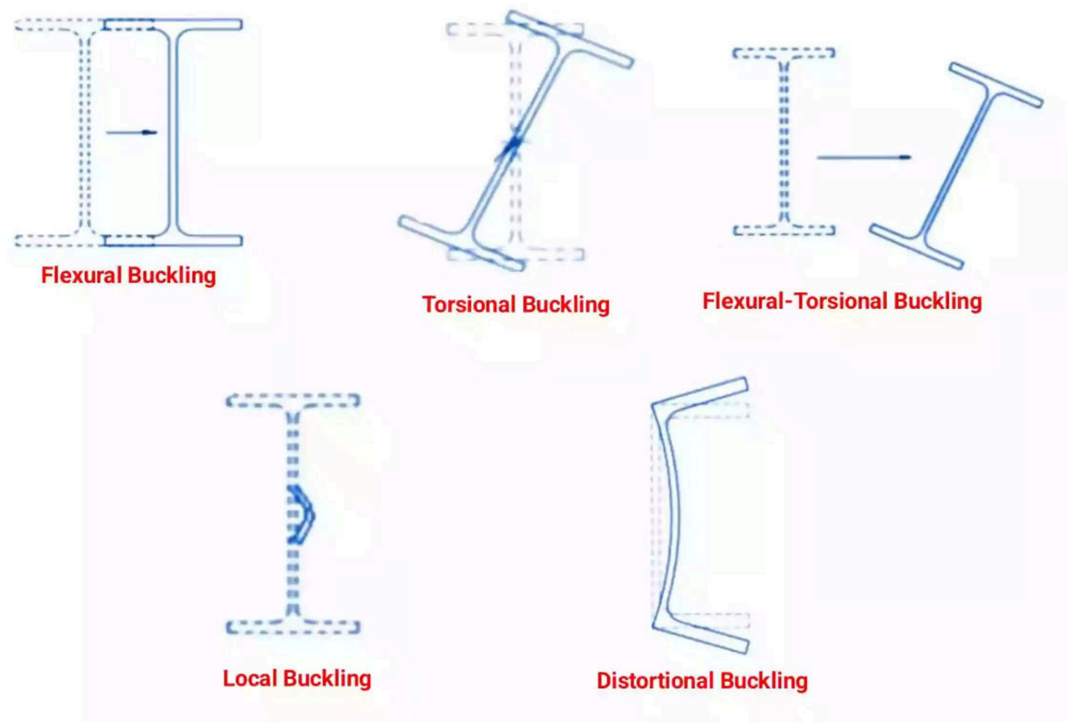
Rigid joints

A third method of providing lateral stability is simply to make the joints strong and stiff enough that movement of the beams relative to the columns is not possible. The black blobs in Fig. indicate stiff joints.

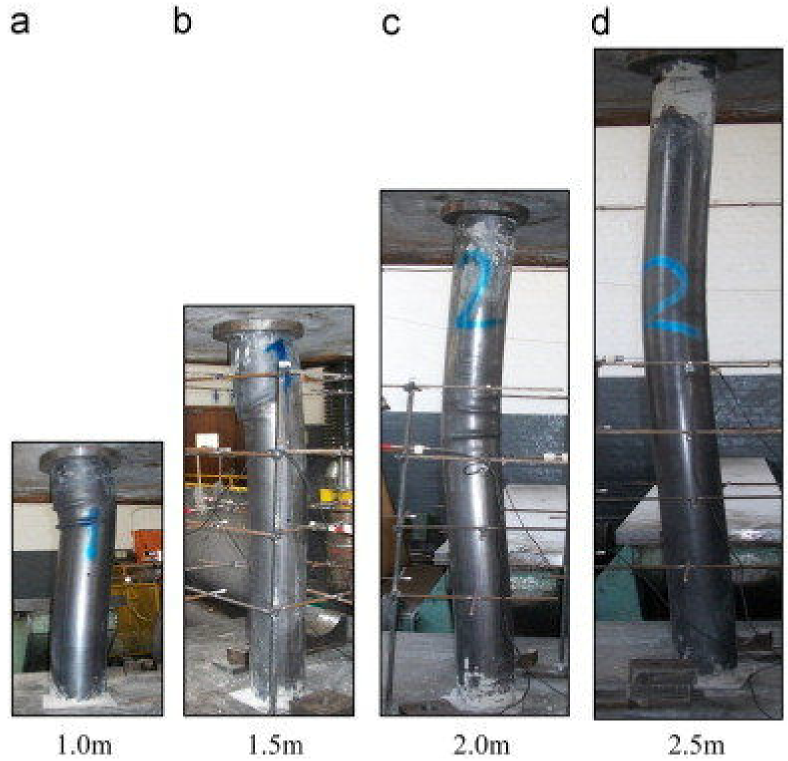


Buckling

- Buckling is characterized by the sudden sideways failure of a structural member that is subject to high compressive stress, where the compressive stress at the point of failure is less than the ultimate compressive stress that the material is able to bear.



Types of Buckling

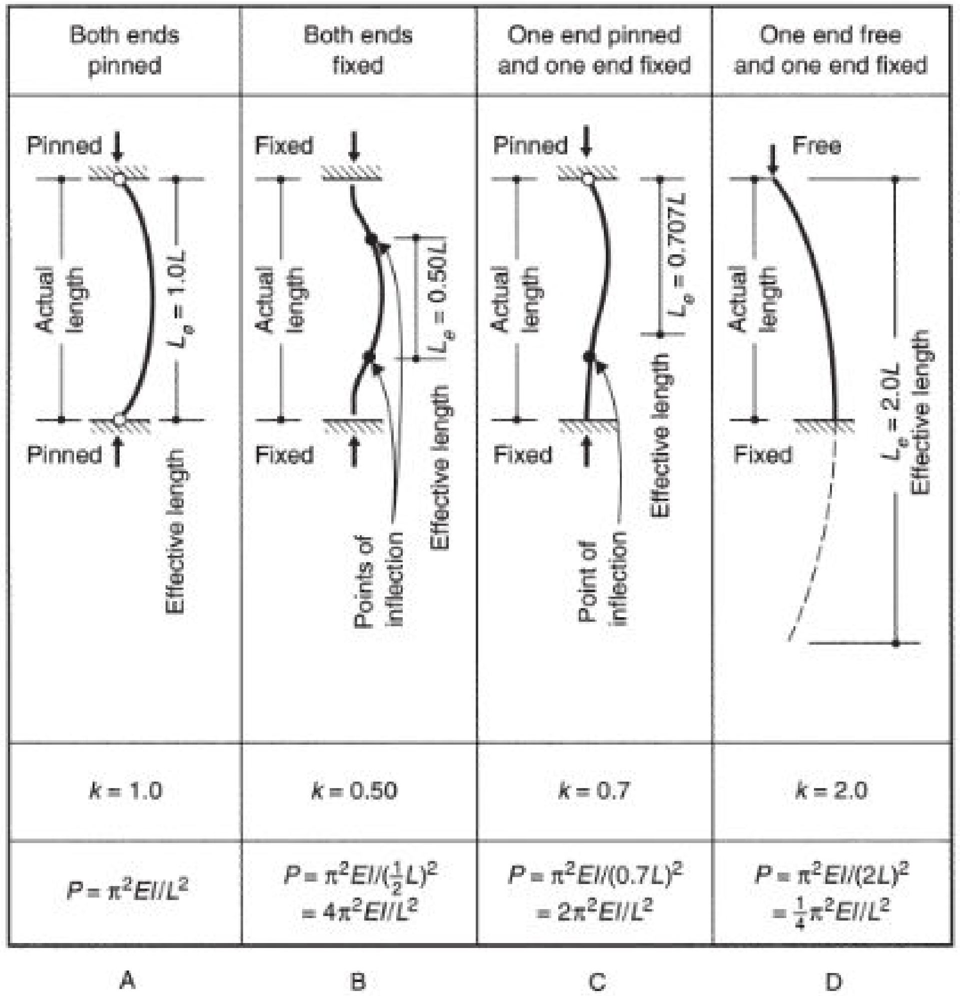


Buckling

- Euler's critical load is the compressive load at which a slender column will suddenly bend or buckle. It is given by the formula

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

where
P_{cr}= Euler's critical load (longitudinal compression load on column),
E= Young's modulus of the column material,
I = minimum area moment of inertia of the cross section of the column (second moment of area),
L= unsupported length of column,
K= column effective length factor



Buckling

- Euler's critical load is the compressive load at which a slender column will suddenly bend or buckle. It is given by the formula

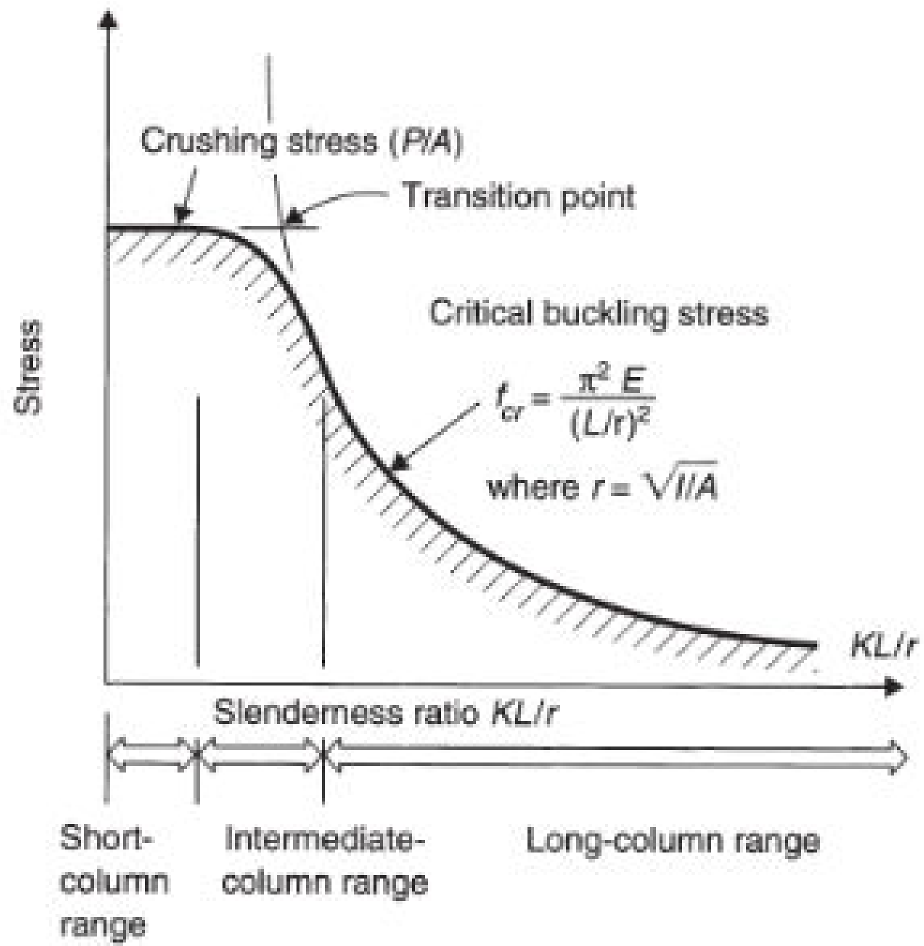
$$\sigma_{all.} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(KL/r)^2}$$

Where:

$\frac{KL}{r}$: the slenderness ratio.

$r = \sqrt{I/A}$ - radius of gyration.

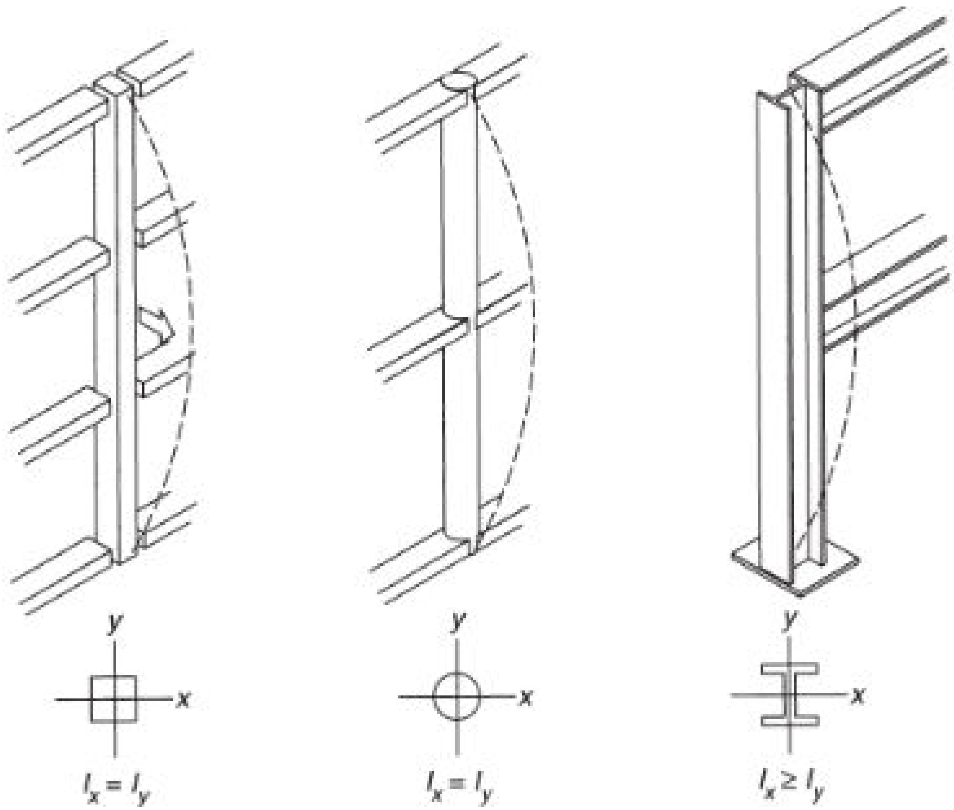
A: the section area.

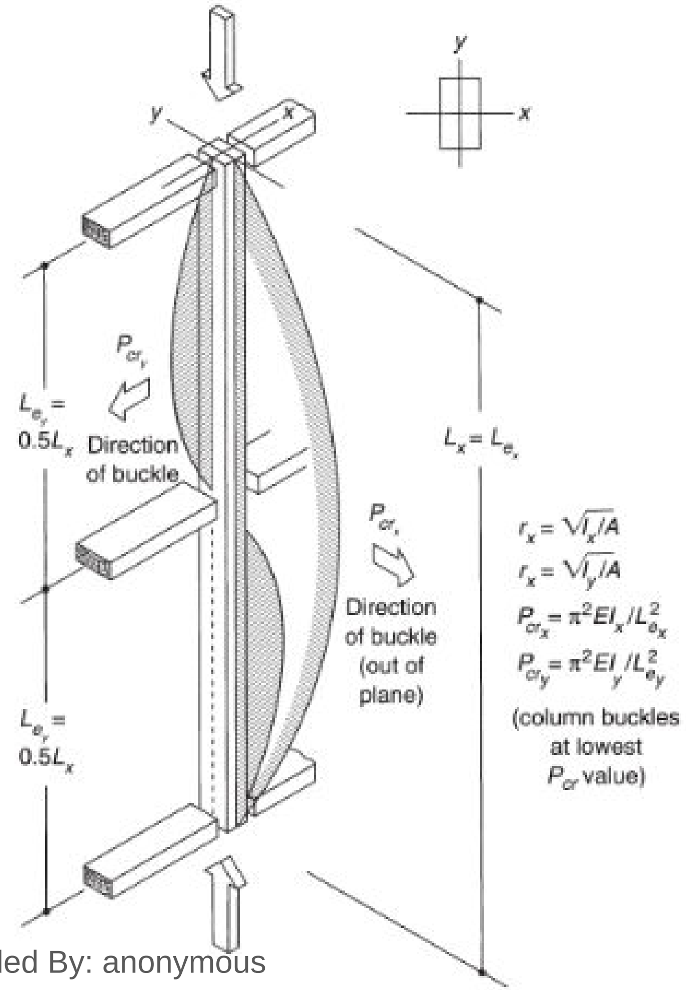


Stress versus length of column

Buckling

- Column bracing in one plane only. When a column is braced in only one plane, it can buckle in two modes. the column will buckle in the mode associated with the higher slenderness ratio (KL/r)



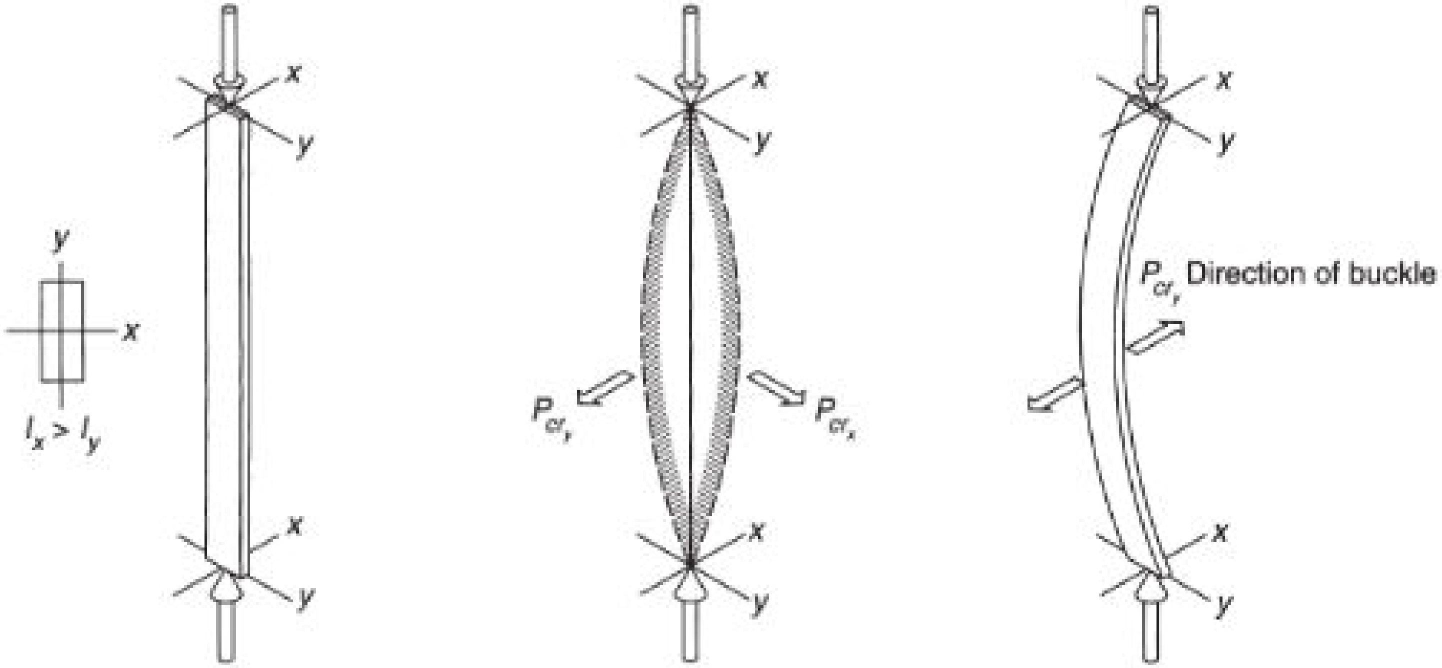


Buckling

Example 1: Determine the critical buckling load for a 50 x 50 mm steel column that is 4.5 m long and pin-ended. Assume that E = 204,000 N/ mm^2.

Buckling

Example 2: Determine the critical buckling load for a rectangular column b = 25 mm and d = 100 mm. Assume that L = 4.5 m; pin-ended; E = 200,000 N/ mm^2.



(a) The moment of inertia about one axis is greater than that about the other.

(b) The member can potentially fail by buckling about either axis. The load required to cause it to buckle about the stronger axis, however, exceeds the load that will cause buckling about the weaker axis.

(c) Consequently, the member will buckle at $P_{cr_y} = \pi^2 EI_y/L^2$ in the mode shown.