

Marginal Revenue

Total Revenue  $R(x)$ Marginal Revenue  $R'(x) = \overline{MR}$ Remark $R'(c)$ : Estimated Revenue for item  $(c+1)$  $C'(c)$ : Estimated Cost for item  $(c+1)$  $P'(c)$ : Estimated profit for item  $(c+1)$ Total Cost  $C(x) \Rightarrow$  Marginal Cost is  $\overline{MC} = C'(x)$ Total profit  $P(x) \Rightarrow$  Marginal profit =  $\overline{MP} = P'(x)$ True Revenue = Exact Revenue =  $R(c+1) - R(c)$ True Cost = Exact Cost =  $C(c+1) - C(c)$ True profit = Exact profit =  $P(c+1) - P(c)$ 

Estimated - Exact

$$\Delta y = y_2 - y_1$$

$$\Delta x = x_2 - x_1$$

$$\Delta R = R_2 - R_1$$

Estimated

Exact

$$\hat{R}(c) \approx DR$$

$$\hat{C}(c) \approx DC$$

$$\hat{P}(c) \approx DP$$

exact  
revenue

$$\Delta R = R_2 - R_1$$

$$= R(c+1) - R(c)$$

exact  
cost

$$\Delta C = C_2 - C_1$$

$$= C(c+1) - C(c)$$

exact  
profit

$$\Delta P = P_2 - P_1$$

$$= P(c+1) - P(c)$$

Exp (Revenue and Marginal Revenue)

Assume the demand for a product in a monopoly market is given by

$$P = 16 - 0.02x$$

where x is the number of units and  
P is the price per unit

① Find total revenue function

$$R(x) = Px$$

$$= (16 - 0.02x)x$$

$$= 16x - 0.02x^2$$

② Find marginal revenue for this product

- ② Find marginal revenue  
at  $x = 40$

$$R'(x) = 16 - (0.02)(2)x$$

$$R'(x) = 16 - 0.04x$$

$$\begin{aligned} R'(40) &= 16 - 0.04(40) \\ &= 16 - 1.6 \\ &= \$14.4 \end{aligned}$$

- ③ what does  $R'(40) = \$14.4$  mean?
- Estimated Revenue for item  $c+1$  is \$14.4

- ④ Find the exact revenue for item  $\underline{41}$

$$\begin{aligned} R(c+1) - R(c) \\ R(41) - R(40) \\ 622.38 - 608 \\ = \$14.38 \end{aligned}$$

$$\begin{aligned} R(x) &= 16x - 0.02x^2 \\ R(40) &= 16(40) - 0.02(40)^2 \\ &= 640 - 32 \\ &= 608 \text{ dollars} \\ R(41) &= 16(41) - 0.02(41)^2 \\ &= 656 - 33.62 \end{aligned}$$

$$= \$14.38$$

$$= 656 - 33.62$$

$$= \$622.38$$

⑤ Find error

$$E = | \text{True} - \text{Estimated} |$$

$$= | 14.38 - 14.4 |$$

$$= 0.02$$

⇒ This means

$R'(40)$  is good approximate for  
 $R(41) - R(40)$

$$\underline{R'(40)} \approx \underline{R(41) - R(40)}$$

estimated  
revenue

$$(14.4)$$

exact revenue

$$(14.38)$$

Exp Suppose the daily total cost in dollars for  
 a certain factory to produce  $x$  kitchen blenders is  
 Marginal Cost  $C(x)$  give exp

$$C(x) = 0.001x^3 - 0.3x^2 + 32x + 2500$$

— is a marginal cost function for these blenders



① Find the marginal cost function for these blenders

$$\overline{MC} = C'(x) = (0.001)(3)x^2 - (0.3)(2)x + 32$$

$$C'(x) = 0.003x^2 - 0.6x + 32$$

② Estimate the exact cost for item  $x=81$

$$C(c) = C(80)$$

$$\begin{array}{r} c+1 = 81 \\ -1 \\ \hline c = 80 \end{array}$$

$$= 0.003(80)^2 - 0.6(80) + 32$$

$$= 19.2 - 48 + 32$$

$$= \$3.2$$

③ Exact cost for item 81  $\rightarrow c+1 = 81$   
 $c = 80$

$$C(c+1) - C(c)$$

$$C(81) - C(80)$$

$$3655.141 - 3652$$

$$= \$3.141$$

$$\begin{aligned} C(x) &= 0.001x^3 - 0.3x^2 + 32x + 2500 \\ C(81) &= 0.001(81)^3 - 0.3(81)^2 + 32(81) + 2500 \\ &= 531.441 - 1968.3 + 2592 + 2500 \\ &= \$3655.141 \end{aligned}$$

$$C(80) = 0.001(80)^3 - 0.3(80)^2 + 32(80) + 2500$$

$$= 512 - 1920 + 2560 + 2500$$

$$= \$3652$$

④ Find and interpret the marginal cost when  $x=80$

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$$= \$ 5076$$

$$✓ \quad C'(80) = \$ 3.2$$

This means the approximate cost for item 81 is \$3.2

⑤ Find  $R(81) - R(80)$ . Explain what does it mean?

$$R(81) - R(80) = \$ 3.141$$

This means the exact cost for item 81 is \$3.141

$$\begin{aligned} \text{⑥ Error} &\Rightarrow E = |\text{True} - \text{Estimated}| \\ &= |3.141 - 3.2| \\ &= 0.059 \end{aligned}$$

Exp (Marginal Profit)

The total profit, thousands of dollars, for a product is given by

$$p(x) = 20\sqrt{x+1} - 2x - 22$$

① what is the marginal profit at production level of 15 units.

$$p'(15)$$

$$\frac{1}{2}$$

$$= 10$$

$$20 \times 10 - 22$$

$$P(x) = 20(x+1)^{\frac{1}{2}} - 2x - 22$$

$$\begin{aligned}\overline{MP} = P'(x) &= 20\left(\frac{1}{2}\right)(x+1)^{-\frac{1}{2}} - 2 \\ &= 10 \frac{1}{(x+1)^{\frac{1}{2}}} - 2\end{aligned}$$

$$\overline{MP} = P'(x) = \frac{10}{\sqrt{x+1}} - 2$$

$$P'(15) = \frac{10}{\sqrt{15+1}} - 2 = \frac{10}{\sqrt{16}} - 2 = \frac{10}{4} - 2 = \frac{5}{2} - 2$$

$$= \frac{5}{2} - 2$$

$$= 2.5 - 2$$

$$P'(15) = 0.5$$

② What does  $P'(15) = 0.5$  mean?

Estimate profit for item 16 is \$ 0.5 thousand

\$ \frac{1}{2} 1000

\$ 500

\$ 500

③ Find Exact profit

$$P(c+1) - P(c)$$

$$P(16) - P(15)$$

$$28.462 - 28$$

$$= 0.462 \text{ dollars in thousands}$$

$$= 0.462 \times 1000$$

$$= 462 \text{ dollars}$$

$$P(x) = 20\sqrt{x+1} - 2x - 22$$

$$P(15) = 20\sqrt{15+1} - 2(15) - 22$$

$$= 20\sqrt{16} - 30 - 22$$

$$= 20(4) - 52$$

$$= 80 - 52$$

$$= 28$$

$$P(16) = 20\sqrt{16+1} - 2(16) - 22$$

$$= 20\sqrt{17} - 32 - 22$$

$$= 82.462 - 54$$

$$= 28.462$$

④ Error

$$E = |T - E| = |462 - 500| = \$38$$

$$P'(15) \approx P(16) - P(15)$$

Estimated  
profit

500

exact  
profit

462



## Exp (Profit in Competitive Market)

A firm in a competitive market must sell its product for \$200 per unit. The cost per unit (per month) is  $80 + x$ , where  $x$ : the number of units sold per month

① Find the marginal profit function  
 $P'(x)$

Total Profit  $P(x) = R(x) - C(x)$

$= P \underset{\substack{\text{دلالة} \\ \text{ibads}}}{x} - (80 + x) \underset{\substack{\text{دلالة} \\ \text{ibads}}}{x}$

$= 200x - (80x + x^2)$

$= 200x - 80x - x^2$

$$P(x) = 120x - x^2$$

$$\overline{MP} = P'(x) = 120 - 2x$$

② Find  $P(10)$ ,  $P'(10)$

$$\begin{aligned} P(10) &= 170(10) - (10)^2 \\ &= 1200 - 100 \\ &= \$1100 \end{aligned}$$

$$\begin{aligned} P'(10) &= 120 - 2(10) \\ &= 120 - 20 \\ &= \$100 \end{aligned}$$

Estimated profit for ibad 11

③ Find exact profit for ibad 11

$$P(11) - P(10)$$

$$1199 - 1100$$

$$\$99$$

$$\begin{aligned} P(11) &= 120(11) - (11)^2 \\ &= 1320 - 121 \\ &= \$1199 \end{aligned}$$

Estimated profit  $P'(10) = \$100$

Exact profit  $P(11) - P(10) = \$99$

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Exp Suppose the total Revenue function is

$$R(x) = 46x$$

and the total cost function

$$C(x) = 100 + 30x + \frac{x^2}{10}$$

$$C(x) = 100 + 30x + \frac{x^2}{10}$$

① Find  $P(100)$

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 46x - \left(100 + 30x + \frac{x^2}{10}\right) \\ &= 46x - 100 - 30x - \frac{x^2}{10} \end{aligned}$$

$$P(x) = 16x - 100 - \frac{x^2}{10}$$

$$\underline{P(100)} = 16(100) - 100 - \frac{(100)^2}{10}$$

$$= 1600 - 100 - \frac{10000}{10}$$

$$= 1500 - 1000$$

$$= \underline{\$500}$$

② Find marginal profit

$$P(x) = 16x - 100 - \frac{x^2}{10}$$

$$\overline{MP} = P'(x) = 16 - 0 - \frac{2}{10}x$$

$$\boxed{P'(x) = 16 - \frac{x}{5}}$$

$$P(x) = 10^{-5}$$

③ Find  $\overline{MP}$  at  $x=100$  and explain what it predicts

$$\overline{MP} = P(100) = 16 - \frac{100}{5}$$

$$= 16 - 20$$

$$= -4 \text{ dollars}$$

Estimated

~~profit~~  
loss

of item 101 is  
4 dollars

④ Find  $P(101) - P(100)$   
value represents

$$P(x) = 16x - 100 - \frac{x^2}{10}$$

and explain what this

$$\begin{aligned} P(100) &= 500 \$ \\ P(101) &= 16(101) - 100 - \frac{101^2}{10} \\ &= 1616 - 100 - \frac{10201}{10} \\ &= 1516 - 1020.1 \\ &= 495.9 \$ \end{aligned}$$

$$\begin{aligned} P(101) - P(100) &= 495.9 - 500 \\ &= -4.1 \text{ dollars} \end{aligned}$$

This means the exact ~~profit~~ loss for item 101 is  
4.1 dollars

⑤ Error

$$E = |T - E| = |-4.1 - (-4)|$$

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$$= |-4.1 + 4|$$

$$= |-0.1|$$

$$= 0.1$$