Row 1: 99.1875

Integer: 99

	÷2
1	99
1	49
0	24
0	12
0	6
1	3
1	1
	0

Fraction: 0.1875

$0.1875 \times 2 = 0.375$

$0.375 \times 2 = 0.75$

$0.75 \times 2 = 1.5$

$0.5 \times 2 = 1.0 \leftarrow$ Stop

$\therefore 99.1875)_{10} = 1100011.0011)_2$

$001100011.001100)_2 = 143.14)_8$

$01100011.00 \ldots .3)_{16}$

Row 2 $\quad 1001 \ldots$

$= 2^6 + 2^3 + 2^1 + 2^{-1} + 2^{-2} + 2^{-4}$

$= 74.8125)_{10}$

$\approx 74.813)_{10}$

$001001011.110100 = 113.64)_8$

$01001011.1101 = 4B.D)_{16}$

Row 3 $\quad 174.564)_8 = 1\times8^2 + 7\times8^1 + 4\times8^0 + 5\times8^{-1} + 6\times8^{-2} + 4\times8^{-3}$

$= 124.7265$

$\approx 124.727)_{10}$

$174.564)_8 = 001111100.101110100)_2$

$= 7C.BA)_{16}$

1, contd.

Row 4: $2BC.7D)_{16} = 2\times16^{2} + 11\times16^{1} + 12 + 7\times16^{-1} + 13\times16^{-2}$

$= 700.48828)_{10}$

$= 700.488)_{10}$

$2BC.7D)_{16} = \underbrace{0010}\,\underbrace{1011}\,\underbrace{1100}.\underbrace{0111}\,\underbrace{1101})_2$

$= 1274.372)_8$

Decimal	Binary	Octal	Hexadecimal
99.1875	1100011.0011	143.14	63.3
74.813	1001011.1101	113.64	4B.D
124.727	1111100.1011101	174.564	7C.BA
700.488	1010111100.01111101	1274.372	2BC.7D

2. 75.8

a. i. $75.8)_{10} = 0111\ 0101.1000)_{BCD}$

ii. ÷2

1	75
1	37
0	18
1	9
0	4
0	2
1	1
	0

$0.8\times2 = 1.6$

$0.6\times2 = 1.2$

$0.2\times2 = 0.4$

$0.4\times2 = 0.8$

$0.8\times2 = 1.6$

$0.8)_{10} = 0.1101$

$75.8)_{10} = \underbrace{0100}\,\underbrace{1011}.\underbrace{1101})_2$

iii. $= 4B.D$

b. ASCII representation

From the ASCII code table on slide 35: Hex:

'7' = 0110111 → with odd parity = 0|0110111 = 37

'5' = 0110101 → 〃 〃 = 1|0110101 = B5

'.' = 0101110 → 〃 〃 = 1|0101110 = AE

'8' = 0111000 → 〃 〃 = 0|0111000 = 38

'75.8' = 37 B5 AE 38

3.(i) $2^n \geq 5639$

$n \log_{10} 2 \geq \log_{10} 5639 \Rightarrow n \geq \frac{3.751}{0.301} \geq 12.46$

$\therefore n_{min} = 13$

(ii) # of additional items $= 2^{13} - 5639$

$= 8192 - 5639 = 2553$

(iii) After 4 years,

Doubling twice $\Rightarrow \underbrace{5639}_{13 \text{ bits}} \times \underbrace{2^2}_{2 \text{ bits}}$

13 bits + 2 bits = 15 bits

4. 21 bits

a. → Max binary codes : $2^{21} = 2$ MB

→ Max # of BCD digits, each BCD digits = 4 bits

$= \text{FLOOR}(21/4) = 5$ digits

→ Max # of 7-bits ASCII codes

$= \text{FLOOR}(21/3) = 7$ codes.

b. n-bits (Integer) Signed-2's complement number

'largest' negative number $= -2^{n-1}$

For $n = 21$ $= -2^{21-1} = -2^{20}$

$= -1{,}048{,}576$

c. n-bit (unsigned) (Fraction) ($n = 21$)

→ Smallest: $2^{-n} = 2^{-21} = 4.768372 \times 10^{-7}$

→ Largest: $1 - 2^{-n} = 9.999995 \times 10^{-1}$

5. unsigned $a = 1010 = 10)_{10}$ $b = 0110 = 6)_{10}$, $c = 0011 = 3)_{10}$

(i) $a + b$ =

```
   1010
 + 0110
  10000  = 16)10
```

(ii) $a - b$ =

```
   1010
 - 0110
   0100  = 4
```

(iii) $a \times b + c$

```
     1010
   x 0110
     0000
    1010
   1010
   111100
 + 0011
   111111 = 63
```

```
  10
 x 6
  60 + 3 = 63
```

(iv) $a \times b - c = 111100 = 60 - 3 = 57$

$$\begin{array}{r} 111100 \\ -000011 \\ \hline 111001 \end{array} = 57$$

6\. i. $16 \times 3 = 54$

$(r+6) \times 3 = (5r+4)$

$3r + 18 = 5r + 4$

$14 = 2r \rightarrow r = 7$, check:

$(7+6) \times 3 = 5 \times 7 + 4$

$39 = 39$

ii. $\frac{44}{3} = 13 \Rightarrow \frac{4r+4}{3} = r+3$

$4r + 4 = 3r + 9 \rightarrow r = 5$

check: $\frac{4 \times 5 + 4}{3} = 5 + 3 = 8$

iii. $14 \times 23 = 410$

$(r+4)(2r+3) = 4r^2 + r$

$2r^2 + 8r + 3r + 12 = 4r^2 + r$

$2r^2 - 10r - 12 = 0$

$(2r+2)(r-6) = 0$

$r = -1$ X or $r = 6$

check: $\underbrace{(6+4)}_{10} \times \underbrace{(2 \times 6 + 3)}_{15} = \underbrace{4(6^2) + 6}_{150}$

7\. 10010110

ASCII code, parity

a. As a 7-bit ASCII code: 1001011 0

From table on slide 35:

$\underbrace{100}\underbrace{1011} = $ 'K'

b. As an unsigned integer: $\overset{76543210}{10010110}$

$= 2^7 + 2^4 + 2^2 + 2^1 = 150)_{10}$

c. signed-1's comp:

$$\underset{\uparrow \text{sign}}{1}0010110 = -\overset{6543210}{01101001}$$

$$= -(2^6 + 2^5 + 2^3 + 2^0)$$

$$= -(64 + 32 + 8 + 1)$$

$$= -105$$

d. signed-2's comp:

$$\underset{\uparrow \text{sign}}{1}0010110 = -01101010$$

$$= -106$$

e. signed-magnitude

$$\underset{\uparrow \text{sign}}{1}\underbrace{0010110}_{\text{mag}} = -00\overset{43210}{10110}$$

$$= -(2^4 + 2^2 + 2^1) = -22$$

f. unsigned BCD:

$$\underbrace{1001}_{9}\ \underbrace{0110}_{6} = 96$$

g. Signed BCD:

$$\underbrace{1001}_{\substack{9 \\ (\text{sign})}}\ 0110 = -6$$

8. -56:

+56 = 0111000

÷2	
0	56
0	28
0	14
1	7
1	3
1	1
	0

(a) In 7-bits

i. -56) signed-mag

= 1111000

ii. -56) signed-1's comp

= 1000111

iii. -56) signed 2's comp

= 1001000

In 12-bits using
b. Sign extension rules

	7-bits	Sign	12-bits
• signed mag	1 | 111000 (sign)	1	00000111000
			sign extension →
• Signed_1's comp	1 | 000111	1	11111000111
			sign extension →
• Signed_2's comp	1 | 001000	1	11111001000

9. unsigned subtraction through conversion to addition

a.
```
  10100     20        10100
- 01101   - 13  →   + 10011
            ---     -------
             7      1←00111 = 7 ✓
```
End carry: ignore, no need for correction

b.
```
  01100     12        01100
- 10110   - 22      + 01010
          -----     -------
          (-)10       10110 → (-)01010
                            = -10)10
```
No end carry: showed correct result by taking its 2's comp & considering it negative

10. a.
```
  101110  = -010010 = -18      -18
- 001110  = +001110 = +14    - +14
                               ---
                               -32
   111
   ←←←
   101110
 + 110010
 --------
 1←100000 = -32 ✓   correct result
```
No overflow

10, contd.

b. 100101 = −011011 ⇒ −27
− 110110 = −001010 ⇒ −−10
−17

100101
+001010
0←0← 101111 → = −010001 = −17 ✓

No overflow

correct result.

c. 111011 = −000101 = −5
+ 001110 = +001110 = + +14
+9

111011
+ 001110 →
1←1← 001001 = +9 ✓ correct result

no overflow

d. 101101 = −010011 = −19
+ 110001 = −001111 = + −15
1←0← 011110 = +30
−34

overflow occured! Wrong Result

more −ive than −32 ∴ overflow Expected

11. (a) +29 in 6-bits
+ −16
+13

011101 → 011101
+ −010000 → + 110000
1← 001101

No overflow

= +13 ✓

correct answer

(b) −17 → −010001 → 101111 → 101111
− +13 → − +001101 → −001101 → + 110011
−30
1←1← 100010

No overflow

100010 = − 011110 = −30

correct result ✓

12. $+145$

$+ \underline{-231}$

a. Using decimal arithmetic

$$\begin{array}{r} +145 \\ + \underline{-231} \end{array} \rightarrow \begin{array}{r} 145 \\ \underline{-231} \\ -086 \end{array}$$

b. Signed-10's complement

$+145 = 0145$

$-231 = -0231 = 10^4 - 0231 = 9769$

$$\therefore \begin{array}{r} +145 \\ + -231 \end{array} \rightarrow \begin{array}{r} 0145 \\ \underline{+\ 9769} \\ 9914 \end{array} = -[10^4 - 9914] = -0086$$

-ive result

correct result

c. Implement in BCD:

	0	1	1	
	0000	0001	0100	0101
+	1001	0111	0110	1001
	1001	1001	1011 (>9)	1110 (>9)
			+ 110	+ 0110
	1001	0 1001	1 0001	1 0100

9914

-ive result

Result $= 10^4 - (9914)$

$= -0086$

correct.