

Rotation

a Rigid body :- a body that can rotate without any change in its shape

a fixed axis :- the rotation occurs about an axis that does not move

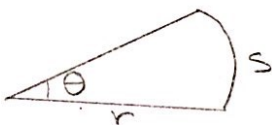
Pure rotation: angular motion

Angular Position

θ : is measured relative to the positive x-axis

$\theta = \frac{s}{r}$ $\rightarrow$ length of the circular arc
 $\rightarrow$ radius of the circle

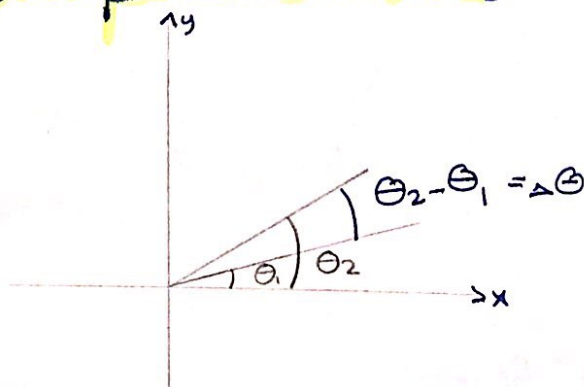
in radians



$$1 \text{ revolution} = 360^\circ = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

Angular displacement

$$\Delta\theta = \theta_2 - \theta_1$$



Alaa Elawad

- $\Delta\theta > 0 \Rightarrow$ if the movement is counter clockwise عقارب الساعة
- $\Delta\theta < 0 \Rightarrow$ if the movement is clockwise مع عقارب الساعة

Angular velocity

$\omega_{average} :-$

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

unit: rad/s
or rev/s

$\omega_{instantaneous}$

$$\omega_{inst} = \frac{d\theta}{dt}$$

Magnitude of ω is called the angular speed

$\omega > 0 \Rightarrow$ if the movement is counter clock wise

$\omega < 0 \Rightarrow$ if the movement is clock wise

Angular acceleration

- If ω is not constant then the body has an angular acceleration

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

unit: rad/s²
or rev/s²

$$\alpha_{inst} = \frac{d\omega}{dt}$$

Maa Etaiw

Are Angular Quantities vectors?

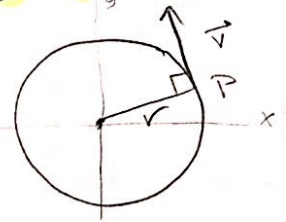
- Yes it is but we don't need to use vector notation
Because we only have 2 cases: Counterclockwise
and we use the plus sign (+) and for clockwise we
use minus sign (-)

• Rotation with Constant Angular acceleration

- When α is constant you can use these three
equations

$$\begin{aligned} 1 - \omega &= \omega_0 + \alpha t \\ 2 - \Delta\theta &= \omega_0 t + \frac{1}{2} \alpha t^2 \\ 3 - \omega^2 &= \omega_0^2 + 2 \alpha (\theta - \theta_0) \end{aligned}$$

• Relating the linear and angular variables



The position $\therefore s = \theta r$

θ : in radians
 ω in rad/s

The speed $\therefore \frac{ds}{dt} = \frac{d\theta}{dt} r$
 $v = \omega r$

- linear speed is always
tangent to the circular
Path

If ω is constant $\rightarrow$ uniform circular motion

$$\begin{aligned} \text{Then } 1 &= \frac{2\pi r}{v} = \frac{2\pi r}{\omega r} \\ &= \frac{2\pi}{\omega} \end{aligned}$$

Also Etanini

The acceleration

$$\frac{dv}{dt} = \frac{d\omega}{dt} r$$

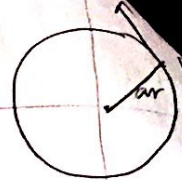
$$a_t = \alpha r$$

in rad/s^2

tangential acceleration

$$a_r = \frac{v^2}{r} = \omega^2 r$$

radial acceleration



Kinetic Energy of Rotation

$$K = \frac{1}{2} I \omega^2$$

$\rightarrow \omega$ is the same for all points
 $\rightarrow$ rotational inertia $I = \sum m_i r_i^2$

I is smaller $\rightarrow$ rotation is easier

- If we have a system of particles then we can calculate I by calculating rotational inertia for each particle in the system:

if we had a system of 2 particles then

$$I = I_1 + I_2$$
$$= m_1 r_1^2 + m_2 r_2^2$$

Knowing that m is the mass and r is perpendicular distance between the particle and the rotation axis

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If we have a continuous system (نظام متصل بحيث انه هناك عدد لا نهائي من الاجسام العفوية (النقاط)) we use integration

$I = \int r^2 dm$ or we use **The parallel axis**

Theory

→ This Theory is used when you have I_{com} (inertia for the Center of Mass) and h (which is the distance between perpendicular the given axis and the axis passing through the Com)

in one condition: The given axis and the axis passing through Com should be **Parallel**

$$\Rightarrow I = I_{com} + Mh^2$$

↳ Mass of the body

Note :-

I :
دالة تسوية
في
ثابت x القوة
X الجذب

I for continuous systems :-

- **Hoop about central axis**

$$I = MR^2$$



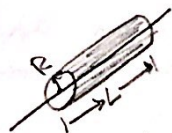
- **Annular cylinder (ring)**

$$I = \frac{1}{2} M (R_1^2 + R_2^2)$$



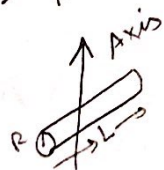
- **Solid cylinder (disk)**

$$I = \frac{1}{2} MR^2$$



- **Solid cylinder (stick)**

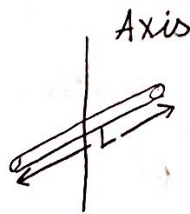
$$I = \frac{1}{12} MR^2 + \frac{1}{12} ML^2$$



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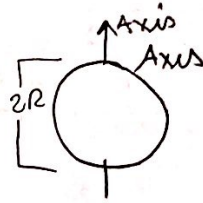
Thin rod about axis

$$I = \frac{1}{12} ML^2$$



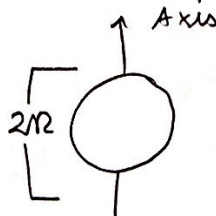
Solid sphere

$$I = \frac{2}{5} MR^2$$



Thin spherical shell

$$I = \frac{2}{3} MR^2$$



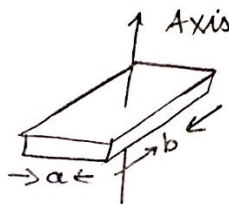
Hoop

$$I = \frac{1}{2} MR^2$$



Slab

$$I = \frac{1}{12} M(a^2 + b^2)$$



Torque (عزم الدوران)

is a product of 2 factors (F and r)

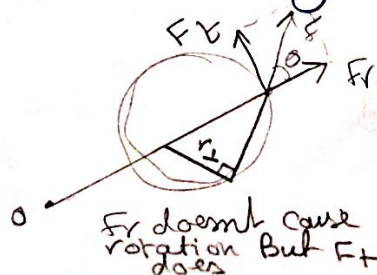
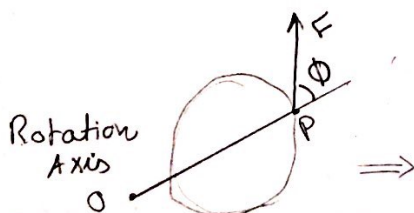
$$\tau = F r \sin \theta$$

$$\text{or } = F_{\perp} r$$

$$\text{or } = r_{\perp} F$$

where $F_{\perp}$ is the tangential component of $\vec{F}$

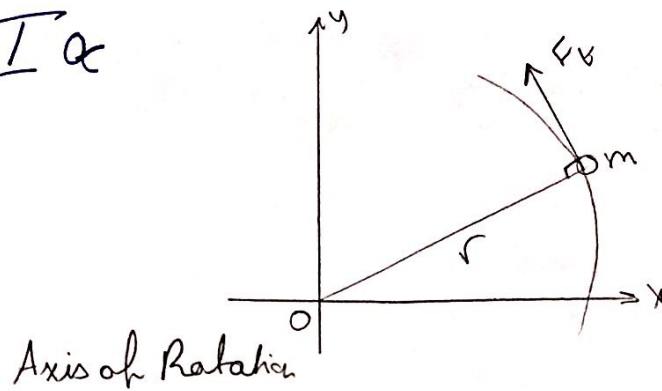
where $r_{\perp}$ is the moment arm (distance between axis of rotation and the extended line running through $\vec{F}$ (Fig 1))



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Newton's second law for Rotation

$$\tau_{\text{net}} = I \alpha$$



Work & Rotational Kinetic Energy

→ Work

$$\begin{aligned} W &= \Delta K \\ &= \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2 \end{aligned}$$

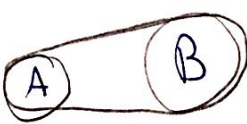
Angular velocity

$$\text{or } \underline{W} = \underline{I}(\Theta_f - \Theta_i)$$

$$\text{and } \underline{P} \text{ (Power)} = \underline{I} \omega$$

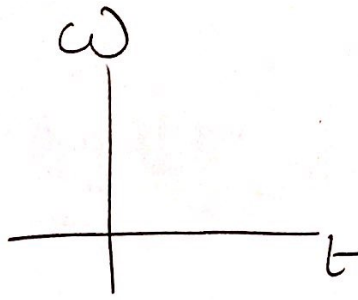
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How to solve Problems

- 1- If you had $\theta(t)$ you can find angular velocity (ω) ^{and α} By differentiation
- 2- If the question gives you number of revolutions it equals θ and you can turn it into radians by multiplying it $\cdot (2\pi)$
- 3- If the question asks for "average" acceleration or velocity you use $\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$, $\omega_{avg} = \frac{\Delta\theta}{\Delta t}$
- 4- always pay attention to units because it helps you to know if you're steps are right or wrong
- 5- θ_{max} can be found using $\Delta\theta = \theta_{max} - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$ if α is constant
- 6- If you're given v (speed) you can turn it into angular velocity using $\omega = \frac{v}{r}$
- 7- $v_A = v_B$ (careful v not ω)  (F)
if the question tells you that the belt does not slip

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8- In these graphs
 α is the slope



9- In questions that has particles and rods with mass

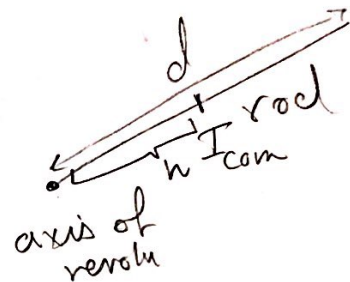
$$I = I_{\text{particle}_1} + I_{\text{particle}_2} + I_{\text{rod}_1} + I_{\text{rod}_2}$$

$$I_{\text{particle}} = md^2$$

I_{rod} : (you have to use The parallel axis Theory)

meaning:- $I_{\text{rod}} = I_{\text{com}} + Mh^2$

$$= \frac{1}{12} Ml^2 + M\left(\frac{l}{2}\right)^2$$



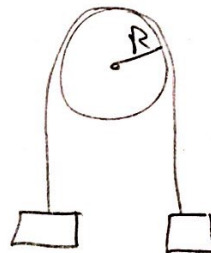
10- When the question asks for the Torque
 pay attention to θ (It's not always the given angle)

11- In such Questions
 at for the pulley = a of the box

12- If you're given F as a function
 you can find T and then α
 using

$$T = Fr$$

$$\text{and } \alpha = \frac{T}{I}$$



13- you can use the Theory Conservation of Energy Theory
 In a lot of questions just pay attention!

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