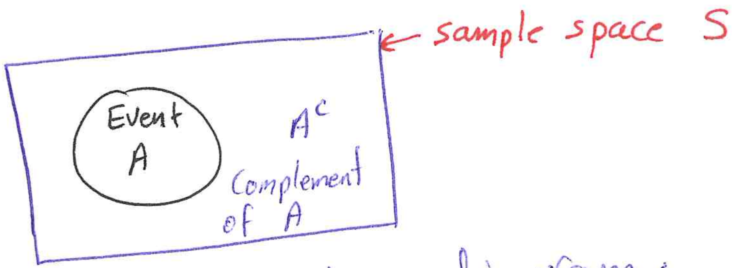
4.3 Some Basic Relationships of Probability

Given an event A. The complement of A, denoted by A^c, consists of all sample points that are not in A.

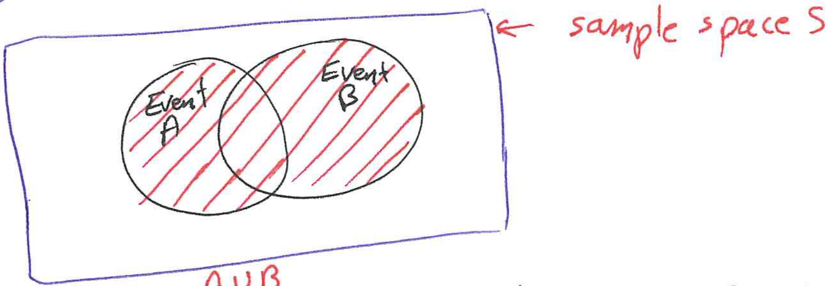
$P(S) = 1$

$P(A) + P(A^c) = 1$



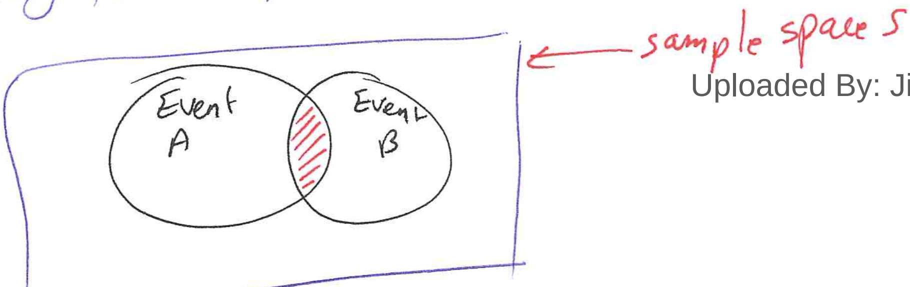
The figure above is called Venn diagram.

- Given the events A and B.
  - The union of A and B is the event containing all sample points belonging to A or B or Both. Denoted by $A \cup B$.



$A \cup B$

- The intersection of A and B, denoted by $A \cap B$, is the event containing the sample points belonging to both A and B.



$A \cap B$

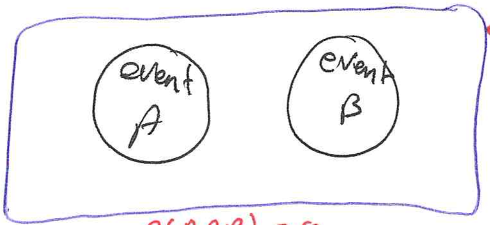
- Addition law:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

* Two events A and B are said to be متنافية **mutually exclusive** if the events A and B have no sample points in common ($P(A \cap B) = 0$).

* Adition law for **mutually exclusive** events :

$$P(A \cup B) = P(A) + P(B)$$



sample space

$P(A \cap B) = 0$

Example: (Q23 page 161) Suppose we have a sample space

$S = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7\}$ where E_i are the sample points.

Given $P(E_1) = P(E_7) = 0.05$, $P(E_2) = P(E_3) = 0.20$

$P(E_4) = 0.25$, $P(E_5) = 0.15$, $P(E_6) = 0.10$.

Let $A = \{E_1, E_4, E_6\}$, $B = \{E_2, E_4, E_7\}$, $C = \{E_2, E_3, E_5, E_7\}$.

[a] Find $P(A)$, $P(B)$, $P(C)$

$P(A) = P(E_1) + P(E_4) + P(E_6) = 0.05 + 0.25 + 0.1 = 0.4$

$P(B) = P(E_2) + P(E_4) + P(E_7) = 0.2 + 0.25 + 0.05 = 0.50$

$P(C) = P(E_2) + P(E_3) + P(E_5) + P(E_7) = 0.20 + 0.20 + 0.15 + 0.05 = 0.60$

[b] Find $A \cup B$ and $P(A \cup B)$

$A \cup B = \{E_1, E_2, E_4, E_6, E_7\}$

$P(A \cup B) = P(E_1) + P(E_2) + P(E_4) + P(E_6) + P(E_7) = 0.05 + 0.20 + 0.25 + 0.10 + 0.05 = 0.65$

[c] Find $A \cap B$ and $P(A \cap B)$

$A \cap B = \{E_4\}$, $P(A \cap B) = P(E_4) = 0.25$

[d] Are the events A and C mutually exclusive?

Yes, they are mutually exclusive because $P(A \cap C) = 0$

[e] Find B^c and $P(B^c)$

$B^c = \{E_1, E_3, E_5, E_6\} \Rightarrow P(B^c) = 1 - P(B) = 1 - 0.5 = 0.5$