

Integration

Def A function $F(x)$ is called Antiderivative of function $f(x)$ on interval I if

$$F'(x) = f(x) \quad \forall x \in I$$

Exo Find 3 antiderivatives for $f(x) = 2x$

$$F_1(x) = x^2 + 2$$

$$F_2(x) = x^2 - \frac{1}{2}$$

$$F_3(x) = x^2 + \pi$$

$$F_4$$

$$F_5$$

$$\vdots$$

$$F(x) = x^2 + C$$

التي هي في الحود
Indefinite Integral

The set of all
anti derivatives

Exo $\int \underbrace{3x^2}_{f(x)} dx = 3 \frac{x^3}{3} + C = \underbrace{x^3 + C}_{F(x)}$

✓ $f(x)$

~~③~~

$F(x)$

$$F(x) = \int f(x) dx$$

$$\boxed{1} \int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int x^4 dx = \frac{x^5}{5} + C$$

$$\begin{aligned} \int (2x' - 5x^2 + 4) dx &= \int (2x' - 5x^2 + 4x^0) dx \\ &= 2 \frac{x^2}{2} - 5 \frac{x^3}{3} + 4 \frac{x^1}{1} + C \\ &= x^2 - \frac{5}{3} x^3 + 4x + C \end{aligned}$$

$$\boxed{2} \int \sin x dx = -\cos x + C$$

$$\boxed{3} \int \cos x dx = \sin x + C$$

$$\boxed{3} \int \sec^2 x dx = \tan x + C$$

$$\textcircled{4} \int \csc^2 x \, dx = -\cot x + C$$

$$\textcircled{5} \int \sec x \tan x \, dx = \sec x + C$$

$$\textcircled{6} \int \csc x \cot x \, dx = -\csc x + C$$

Exp Find

$$\textcircled{1} \int (\bar{x}^3 - 3x^2 + \sqrt{2}) \, dx = \frac{x^{-2}}{-2} - 3 \frac{x^3}{3} + \sqrt{2}x + C$$

$$= -\frac{1}{2x^2} - x^3 + \sqrt{2}x + C$$

$$\textcircled{2} \int \underline{\cos^2 x} \, dx$$

$$\int \frac{1 + \cos 2x}{2} \, dx$$

$$\int \left(\frac{1}{2} + \frac{1}{2} \cos 2x \right) dx$$

$$\frac{1}{2}x + \frac{1}{2} \int \underline{\cos 2x} \, dx$$

$$\frac{x}{2} + \frac{1}{2} \left(\frac{\sin 2x}{2} \right) + C$$

$$\begin{aligned} \cos 2x &= 2 \cos^2 x - 1 \\ &= 1 - 2 \sin^2 x \\ &= \cos^2 x - \sin^2 x \end{aligned}$$

$$\cos 2x + 1 = 2 \cos^2 x$$

$$\boxed{\frac{1 + \cos 2x}{2} = \underline{\underline{\cos^2 x}}}$$

$$2 \sin^2 x = 1 - \cos 2x$$

$$\frac{1}{2} \begin{matrix} \cos(2x) \rightarrow 2 \\ \cos 2x \end{matrix}$$

$$\frac{x}{2} + \frac{1}{2} \left(\frac{2}{2} \right) + C$$

$$\frac{x}{2} + \frac{\sin 2x}{4} + C$$



(correct)

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$(3) \int \sin^2 x \, dx$$

$$\int \frac{1 - \cos 2x}{2} \, dx = \int \left(\frac{1}{2} - \frac{1}{2} \cos 2x \right) dx$$

$$= \frac{1}{2}x - \frac{1}{2} \int \cos 2x \, dx$$

$$= \frac{x}{2} - \frac{1}{2} \frac{\sin 2x}{2} + C$$



$$= \frac{x}{2} - \frac{\sin 2x}{4} + C$$

$$(4) \int \frac{\csc \theta}{\csc \theta - \sin \theta} d\theta$$

$$\int \frac{\frac{1}{\sin \theta}}{\frac{1}{\sin \theta} - \sin \theta} d\theta$$

$$\int \frac{\frac{1}{\sin \theta}}{\frac{1}{\sin \theta} - \frac{\sin^2 \theta}{\sin \theta}} d\theta$$

$$\int \frac{\cancel{\frac{1}{\sin \theta}}}{\frac{1 - \sin^2 \theta}{\cancel{\sin \theta}}} d\theta = \int \frac{d\theta}{1 - \sin^2 \theta}$$

$$\int \frac{d\theta}{\cos^2 \theta} = \int \sec^2 \theta d\theta$$

$$= \tan \theta + C$$

5) $\int \frac{\sec t \tan t}{\sqrt{\sec t}} dt$

$$\int \frac{\sec t \tan t}{\sqrt{\sec t}} \left(\frac{\sqrt{\sec t}}{\sqrt{\sec t}} \right) dt$$

157

v -

$$\frac{\sec t \tan t \sqrt{\sec t}}{\sec t} dt$$

$$\int \frac{\sec t \tan t}{\sqrt{\sec t}} dt = \int \frac{\frac{1}{\cos t} \frac{\sin t}{\cos t}}{\sqrt{\frac{1}{\cos t}}} dt$$

تعويض

$$\int \frac{\sec t \tan t}{\sqrt{\sec t}} dt = \int \sec^{\frac{1}{2}} (\sec t)^{-\frac{1}{2}} \tan t dt$$

$$= \int \sqrt{\sec t} \tan t dt$$

تعويض

$$\int \frac{\sec t \tan t}{\sqrt{\sec t}} dt$$

$$(\sec t)' = \sec t \tan t$$

$$u = \sec t$$

$$du = \sec t \tan t dt$$

$$\int \frac{du}{\sqrt{u}}$$

$$\int u^{-\frac{1}{2}} du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{u} + C$$

$$\int u^{-\frac{1}{2}} du = \frac{u}{\frac{1}{2}} + C = 2\sqrt{u} + C = 2\sqrt{\sec t} + C$$

$\int f(x) dx$ indefinite integral

$\int_a^b f(x) dx$ definite integral
 جملہ (جمع)

Exp

$$\begin{aligned} & \int_0^1 (6x^2 - 4x + 2) dx \\ &= \left. 6 \frac{x^3}{3} - 4 \frac{x^2}{2} + 2x \right|_0^1 \\ &= \left. 2x^3 - 2x^2 + 2x \right|_0^1 \\ &= 2(1)^3 - (2)(1)^2 + (2)(1) - 0 \\ &= 2 - 2 + 2 \\ &= 2 \end{aligned}$$

Exp Find y' if $y = \int_0^3 \frac{\tan^3 x}{1 - \sin^4 x} dx$

$y = \text{const}$
 $y' = 0$

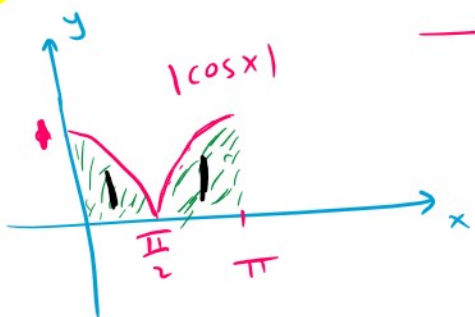
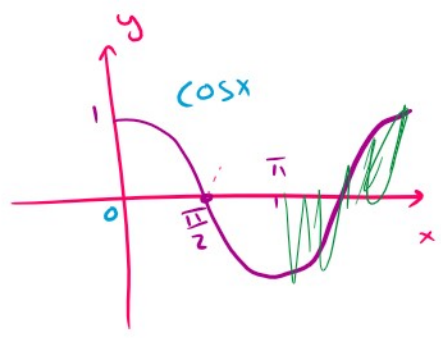
Exp $\int_0^\pi (\cos x + |\cos x|) dx$

$\int_0^\pi \cos x dx + \int_0^\pi |\cos x| dx$

$\int_0^\pi \cos x dx = \sin x \Big|_0^\pi = \sin \pi - \sin 0 = 0 - 0 = 0$

$|\cos x| = \begin{cases} \cos x & \text{if } 0 \leq x \leq \frac{\pi}{2} \\ -\cos x & \text{if } \frac{\pi}{2} \leq x \leq \pi \end{cases}$

$0 + \int_{\frac{\pi}{2}}^{\pi} \cos x dx + \int_0^{\frac{\pi}{2}} -\cos x dx$

$$0 + \int_0^{\frac{\pi}{2}} \cos x \, dx + \int_{\frac{\pi}{2}}^{\pi} -\cos x \, dx$$

$$\sin x \Big|_0^{\frac{\pi}{2}} + -\sin x \Big|_{\frac{\pi}{2}}^{\pi}$$

$$\sin \frac{\pi}{2} - \sin 0 + (-\sin \pi - -\sin \frac{\pi}{2})$$

$$1 - 0 + (0 + 1)$$

$$1 + 1 = 2$$

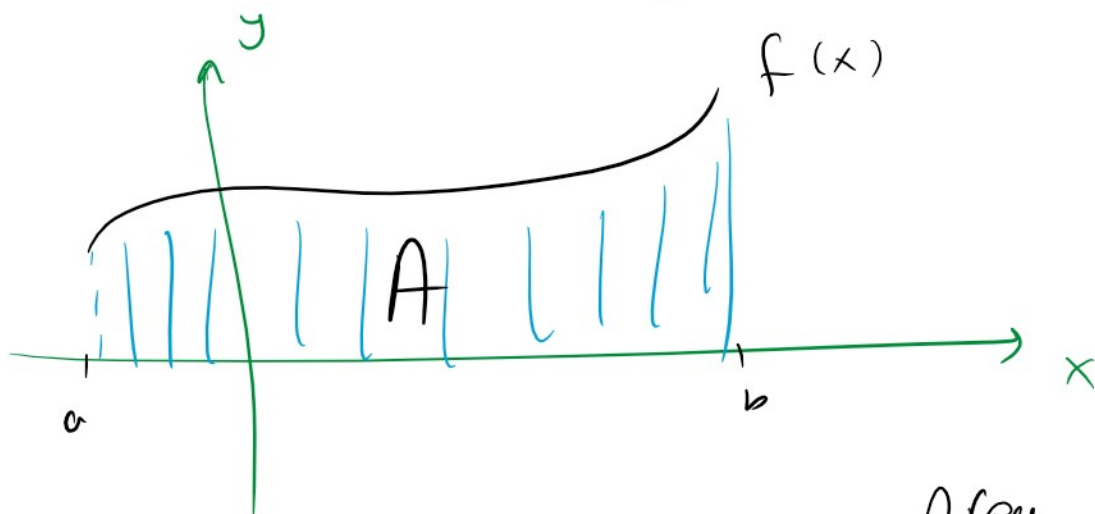
$$\int_a^b f(x) \, dx \longrightarrow \text{قسط}$$

فقط إذا كان $f(x) \geq 0$
Area

If $f(x) \geq 0$ on $I = [a, b]$ then

$$\int_a^b f(x) dx = \text{area} \quad \checkmark \text{ under } f(x) \text{ on } [a, b]$$

= area bounded by f and x -axis



$$\int_a^b f(x) dx = A \quad \text{Area}$$

Exp Find ① $\int_0^4 x dx$

$$= \left. \frac{x^2}{2} \right|_0^4 = \frac{4^2}{2} - \frac{0^2}{2} = \frac{16}{2} = 8$$

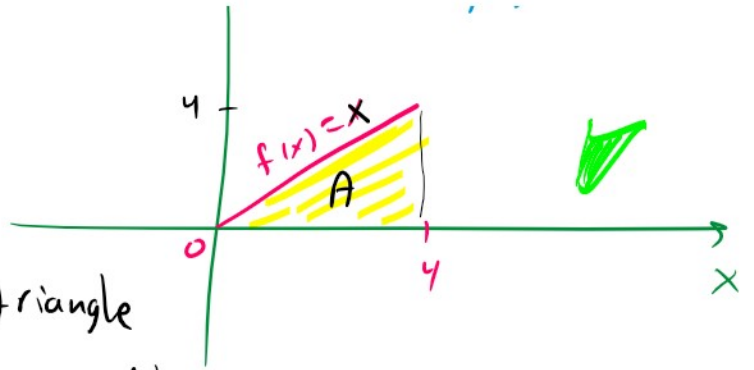
$f(x) = x$ on $[0, 4]$
 $y = x$



$f > 0$ on $[0, 4]$

$$f(x) = x \quad \text{on } [0, 4]$$

$$y = x$$



$$\begin{aligned} A &= \int_0^4 x \, dx = \text{area of triangle} \\ &= \frac{1}{2} (\text{base}) (\text{height}) \\ &= \frac{1}{2} (4) (4) \\ &= 8 \end{aligned}$$