Exercises :

Q 48 : Find the constant c so that f(x) satisfies the condition of being a p.d.f of one Random variable X .

a. $f(x) = \begin{cases} c\left(\frac{2}{3}\right)^x , & x = 1,2,3,\ldots, \\ 0 , & \text{elsewhere} \end{cases}$

Geometric : $\text{sum} = \frac{a}{1-r}$

$r = \frac{4}{9} \div \frac{2}{3} = \frac{4}{9} \times \frac{3}{2} = \frac{2}{3}$

$\rightsquigarrow \sum_{x=1}^{\infty} c\left(\frac{2}{3}\right)^x = 1 \Rightarrow c \sum_{x=1}^{\infty} \left(\frac{2}{3}\right)^x = 1$

$c\left(\frac{\frac{2}{3}}{1-\frac{2}{3}}\right) = 1$

$\frac{2}{3} \div \frac{1}{3} = \frac{2}{3} \times \frac{3}{1} = 2$

$c\,(2) = 1 \rightsquigarrow \boxed{c = \frac{1}{2}}$

b. $f(x) = \begin{cases} cx , & x = 1,2,3,4,5,6 \\ 0 , & \text{elsewhere} \end{cases}$

$\rightsquigarrow \sum_{x=1}^{x=6} cx = 1 \rightsquigarrow c \sum_{x=1}^{x=6} x = 1$

$c\,(1+2+3+4+5+6) = 1$

$21\,c = 1$

$\boxed{c = \frac{1}{21}}$

Q49: let $f(x) = \begin{cases} \frac{x}{15} & , x = 1,2,3,4,5 \\ 0 & , \text{elsewhere} . \end{cases}$ be the p.d.f of X, Find

1. $Pr(X=1 \text{ or } 2) = Pr(X=1) + Pr(X=2)$

$= \frac{1}{15} + \frac{2}{15}$

$= \frac{3}{15} = \frac{1}{5}$

2. $Pr(\frac{1}{2} < X < \frac{5}{2}) = Pr(X=1) + Pr(X=2)$

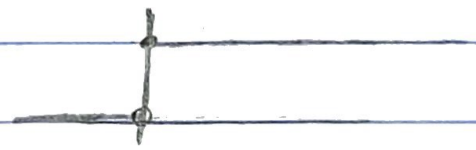
$= \frac{3}{15} = \frac{1}{5}$

3. $Pr(1 \leq X \leq 2) = Pr(X=1) + Pr(X=2)$

$= \frac{1}{5}$

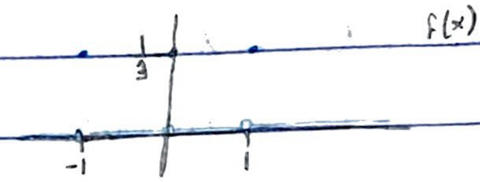
Q50: let f(x) be the p.d.f of a Random variable X. Find the distribution function F(x) of X and sketch its graph . . .

a. $f(x) = \begin{cases} 1 & , x = 0 \\ 0 & , \text{elsewhere} . \end{cases}$

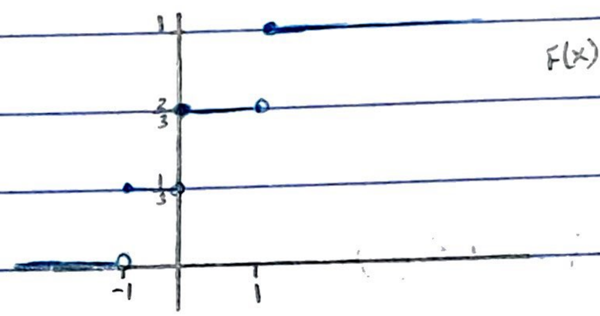


$F(x) = \begin{cases} 0 , & x < 0 \\ 1 , & x \geq 0 \end{cases}$

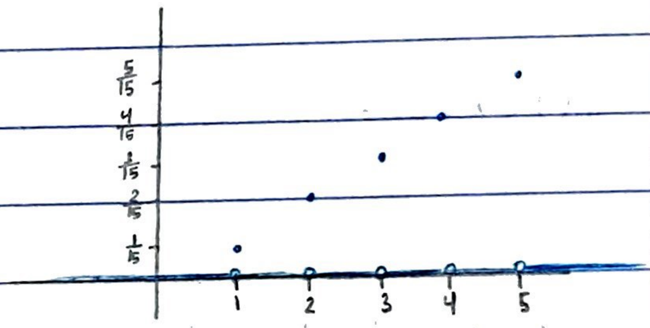
b. $f(x) = \begin{cases} \frac{1}{3}, & x = -1, 0, 1 \\ 0, & \text{elsewhere}. \end{cases}$



$F(x) = \begin{cases} 0, & x < -1 \\ \frac{1}{3}, & -1 \leq x < 0 \\ \frac{2}{3}, & 0 \leq x < 1 \\ 1, & 1 \leq x \end{cases}$



c. $f(x) = \begin{cases} \frac{x}{15}, & x = 1, 2, 3, 4, 5 \\ 0, & \text{else where}. \end{cases}$



$F(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{15}, & 1 \leq x < 2 \\ \frac{3}{15}, & 2 \leq x < 3 \\ \frac{6}{15}, & 3 \leq x < 4 \\ \frac{10}{15}, & 4 \leq x < 5 \\ 1, & 5 \leq x \end{cases}$

برسمه ويكون على شكل درجات

Q53: let X have the p.d.f $f(x) = \begin{cases} \frac{x}{5050} & , x = 1,2,3,\ldots,100 \\ 0 & , \text{elsewhere} \end{cases}$.

a. Compute $Pr(X \leq 50)$.

$$P_r(X \leq 50) = \sum_{x=1}^{x=50} \frac{x}{5050} = \frac{1}{5050} \sum_{x=1}^{x=50} x$$

Note:

$$\sum_{x}^{n} x = \frac{n(n+1)}{2}$$

$$= \frac{1}{5050} \times \frac{50(51)}{2}$$

$$= \frac{2550}{10100} = 0.252$$

b. Show that the distribution function of X is $F(x) = \frac{[x]([x]+1)}{10100}$ for $1 \leq x \leq 100$, where $[x]$ greatest integer in x.

For $x = 1, \ldots, 100$

$$F(x) = P(X \leq x) = \sum_{w=1}^{x} f(w) = \sum_{w=1}^{x} \frac{x}{5050}$$

$$= \frac{1}{5050} \sum_{w=1}^{x} f(w)$$

$$= \frac{1}{5050} \cdot \frac{x(x+1)}{2}$$

Then

$$F(x) = \begin{cases} 0 & , x < 1 \\ \frac{[x] \cdot [x+1]}{10100} & , 0 \leq x \leq 100 \\ 1 & , x > 100 \end{cases}$$

Q55: casts a die a number of independent times until a six appears on the up side of the die.

a. Find the p.d.f. $f(x)$ of X, the number of casts needed to obtain that first six:

$Pr(X=1) = \frac{1}{6}$ ما يطلع من أول مرة 6

$Pr(X=2) = \left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$

$Pr(X=3) = \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$

$Pr(X=4) = \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$

$\vdots$

$$\Rightarrow Pr(X=x) = \left(\frac{5}{6}\right)^{x-1}\left(\frac{1}{6}\right)$$

$$\text{So} \quad f(x) = \begin{cases} \left(\frac{5}{6}\right)^{x-1}\left(\frac{1}{6}\right), & x = 1, 2, 3, \dots \\ 0, & \text{elsewhere} \end{cases}$$

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b. Show that $\sum_{x=1}^{\infty} f(x) = 1$.

$$\sum_{x=1}^{\infty} f(x) = \frac{1}{6}\sum_{x=1}^{\infty}\left(\frac{5}{6}\right)^{x-1} = \frac{1}{6}(6) = 1$$

$a = 1$

$r = \frac{5}{6} \div 1 = \frac{5}{6}$

$1 - r = \frac{1}{6}$

$\frac{a}{1-r} = 1 \div \frac{1}{6} = 6$

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c. Determine $Pr(X = 1, 3, 5, 7, \dots) = \frac{1}{6} + \frac{1}{6}\left(\frac{5}{6}\right)^{3-1} + \frac{1}{6}\left(\frac{5}{6}\right)^{4} + \frac{1}{6}\left(\frac{5}{6}\right)^{6} + \dots$

$$= \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \frac{1}{6}\left(\frac{5}{6}\right)^4 + \dots\right]$$

$$= \frac{1}{6}\sum_{x=0}^{\infty}\left(\frac{5}{6}\right)^{2x}$$

$a = 1$

$r = \frac{25}{36} \div 1 = \frac{25}{36}$

$1 - r = \frac{11}{36}$

$\frac{a}{1-r} = 1 \times \frac{36}{11}$

$$= \frac{1}{6}\left(\frac{36}{11}\right) = \frac{6}{11} \checkmark$$