Chapter 1

chapter 1 : signals

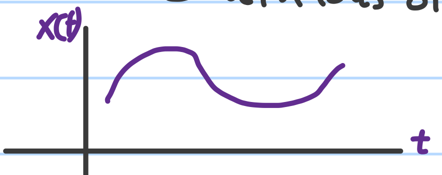
Signals

signal real scenario
- speach
- vidio
- image

Math function
- 1D : speach
- 2D : image
- 3D : vidio

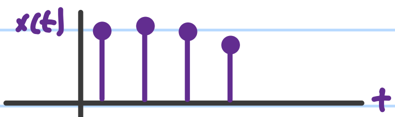
Type of signals

continuous signal



$x(t) \quad -\infty < t < \infty$

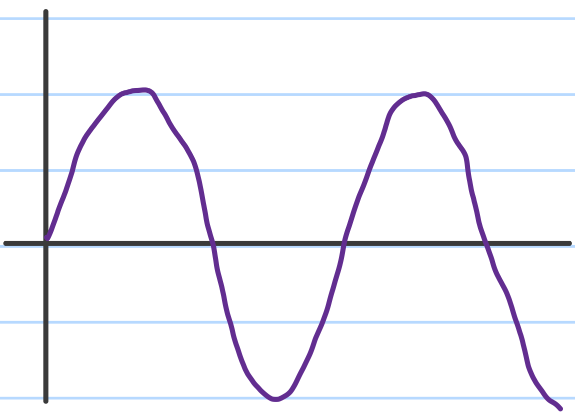
Discrete signal



$x(nT) = x[n]$; $\quad n$: int

- periodic and aperiodic
- Random

Periodic and aperiodic



T_0

where T_0 : fundamental Period [sec]

f_0: fundamental frequency

where $f_0 = \frac{1}{T_0}$ [Hz]

In general

$$x(t) = A \sin(\underbrace{2\pi f_0}_{\text{wo (angular freq.) [rad/sec]}} t + \theta)$$

Amp — A; phase — θ

Remember

$\cos(\alpha \mp \beta) = \cos\alpha \cos\beta \pm \sin\alpha \sin\beta$

$\sin(\alpha \mp \beta) = \sin\alpha \cos\beta \mp \cos\alpha \sin\beta$

$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$ $+$

$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin\alpha \sin(\beta)$ $-$

* $\cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta)$

* $\sin(\alpha)\sin(\beta) = \frac{1}{2}\cos(\alpha-\beta) - \frac{1}{2}\cos(\alpha+\beta)$

from $\sin(\alpha \pm \beta)$:-

$\sin(\alpha)\cos(\beta) = \frac{1}{2}\sin(\alpha+\beta) + \frac{1}{2}\sin(\alpha-\beta)$

$\cos(\alpha)\sin(\beta) = \frac{1}{2}\sin(\alpha+\beta) - \frac{1}{2}\sin(\alpha-\beta)$

- $\cos(\alpha) = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$

- $\sin(\alpha) = \frac{e^{j\alpha} - e^{-j\alpha}}{2}$

Periodic and aperiodic signals

LEC 2

In general

The periodic can be expressed as

$x(t+T_0) = x(t)$

Ex. Check if the following signals are periodic or not :-

① $x(t) = 3+4\cos(30\pi t+\frac{\pi}{4})$

To check if $x_1(t)$ is periodic or not

① $x_1(t+T_0) \overset{??}{=} x_1(t)$

② $x_1(t+T_0) = 3+4\cos(30\pi(t+T_0)+\frac{\pi}{4})$

$$= 3+4\cos(30\pi t+30\pi T_0+\frac{\pi}{4})$$

$$= 3+4\cos(30\pi t+\frac{\pi}{4}+30\pi T_0)$$

$$= 3+4\cos(\underbrace{30\pi t+\frac{\pi}{4}}_{\alpha}+\underbrace{2\pi}_{\beta})$$

* $\omega = 2\pi f_0 = \frac{2\pi}{T_0}$

$\Rightarrow T_0 = \frac{2\pi}{\omega_0}$, $\omega_0 = 30\pi$

Since: $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$x_1(t+T_0) = 3+4[\cos(30\pi t+\frac{\pi}{4})\underbrace{\cos(2\pi)}_{1} - \cancel{\sin(30\pi t+\frac{\pi}{4})\underbrace{\sin(2\pi)}_{0}}]$

$\Rightarrow x(t+T_0) = 3+4\cos(30\pi+\frac{\pi}{4})$

↳ $= x_1(t)$ is periodic signal

Fundamental period $= \frac{2\pi}{\omega_0} = \frac{2\pi}{30\pi} = \frac{1}{15}$ sec

Fundamental freq $= \frac{1}{T_0} = \frac{1}{\frac{1}{15}} = 15$ Hz

* في حال وجود (sin او cos) ω_0 ← عبارة عن معامل (t)

* α ← عبارة عن الموجود في $x(t)$ ، β ← عبارة عن الزيادة

Ex. (2) $3\sin(15t)$

(1) $x_2(t) \overset{??}{=} x(t+T_0)$

(2) $x_2(t+T_0) = 3\sin(15(t+T_0))$

$= 3\sin(15t + 15T_0)$ $\Rightarrow T_0 = \frac{2\pi}{\omega} = \frac{2\pi}{15}$

$= 3\sin(\underbrace{15t}_{\alpha} + \underbrace{2\pi}_{\beta})$

Since: $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$x_2(t+T_0) = 3[\sin(15t)\underbrace{\cos(2\pi)}_{\to 1} + \cos(15t)\cancel{\underbrace{\sin(2\pi)}_{\to 0}}]$

$x_2(t+T_0) = 3\sin(15t)$

↳ $x_2(t)$ is periodic signal

fundamental period $= \frac{2\pi}{\omega_0} = \frac{2\pi}{15}$

fundamental freq. $= \frac{1}{T_0} = \frac{15}{2\pi}$

Ex. (3) $\sin^2(30\pi t + \frac{\pi}{3})$

Remember

Function Relationships

$\sin\theta = \frac{1}{\csc\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Opposite Angle Formulas

$\sin(-\theta) = -\sin(\theta)$

$\cos(-\theta) = \cos(\theta)$

$\tan(-\theta) = -\tan(\theta)$

$\cot(-\theta) = -\cot(\theta)$

$\sec(-\theta) = \sec(\theta)$

$\csc(-\theta) = -\csc(\theta)$

Cofunction Formulas (in Quadrant I)

$\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$ $\cos\theta = \sin\left(\frac{\pi}{2}-\theta\right)$

$\tan\theta = \cot\left(\frac{\pi}{2}-\theta\right)$ $\cot\theta = \tan\left(\frac{\pi}{2}-\theta\right)$

$\sec\theta = \csc\left(\frac{\pi}{2}-\theta\right)$ $\csc\theta = \sec\left(\frac{\pi}{2}-\theta\right)$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

Half Angle Formulas

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

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Angle Addition Formulas

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Double Angle Formulas

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Product-to-Sum Formulas

$\sin A\cdot\sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

$\cos A\cdot\cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

$\sin A\cdot\cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\cos A\cdot\sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$

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Triple Angle Formulas

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

Power Reducing Formulas

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$

Sum-to-Product Formulas

$\sin A + \sin B = 2\cdot\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

$\sin A - \sin B = 2\cdot\sin\left(\frac{A-B}{2}\right)\cdot\cos\left(\frac{A+B}{2}\right)$

$\cos A + \cos B = 2\cdot\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

$\cos A - \cos B = -2\cdot\sin\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$

Arc Length

$S = r\theta$

Law of Sines $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Law of Tangents

$\frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(A-B)\right]}{\tan\left[\frac{1}{2}(A+B)\right]}$

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Mollweide's Formulas

$\frac{a+b}{c} = \frac{\cos\left[\frac{1}{2}(A-B)\right]}{\sin\left(\frac{1}{2}C\right)}$

$\frac{a-b}{c} = \frac{\sin\left[\frac{1}{2}(A-B)\right]}{\cos\left(\frac{1}{2}C\right)}$

Euler's Formula

$e^{i\theta} = \cos\theta + i\sin\theta = \text{cis}\,\theta$

Polar Multiplication and Division

Let: $a = r_1\,\text{cis}\,\theta$ $b = r_2\,\text{cis}\,\varphi$

$a\cdot b = r_1 r_2\,\text{cis}(\theta+\varphi)$ $\frac{a}{b} = \frac{r_1}{r_2}\,\text{cis}(\theta-\varphi)$

DeMoivre's Formula

$(r\,\text{cis}\,\theta)^n = r^n\,\text{cis}(n\theta)$

$\sin^2(30\pi t + \frac{\pi}{3})$

$= \frac{1}{2} - \frac{1}{2}\cos(60\pi t + \frac{2\pi}{3}$

$x(t+T_0) = \frac{1}{2} - \frac{1}{2}\cos(60\pi(t+T_0) + \frac{2\pi}{3})$ $\quad * \; T_0 = \frac{1}{30}$

$= \frac{1}{2} - \frac{1}{2}\cos(\underbrace{60\pi t + \frac{2\pi}{3}}_{\alpha} + \underbrace{60\pi T_0}_{\beta})$

$= \frac{1}{2} - \frac{1}{2}\left[\cos(60\pi t + \frac{2\pi}{3})\underbrace{\cos(2\pi)}_{=1} - \cancel{\sin(60\pi t + \frac{2\pi}{3})\sin(2\pi)}\right]$ 0

$\Rightarrow \frac{1}{2} - \frac{1}{2}\cos(60\pi t + \frac{2\pi}{3})$

$\hookrightarrow x(t) = x(t+T_0)$ is periodic signal

fundamental period $= \frac{1}{30}$ sec

fundamental freq $= 30\,Hz$

→ cont.

Ex. 4 $x(t) = 3\cos(20\pi t) + 4\cos(60\pi t)$

$\omega_1 = 2\pi f_1 = 20\pi$ $\quad\quad$ $\omega_2 = 2\pi f_2 = 60\pi$

$f_1 = 10$ $\quad\quad$ $\omega_2 = 30$

$n_1 f_0 = 10$ $\quad\quad$ $n_2 f_0 = 30$

$$\frac{n_1 f_0}{n_2 f_0} = \frac{10}{30} = \frac{1}{3}$$ → rational number ⇒ periodic signal

Ex 5 $x(t) = 4\sin^2(20\pi(t)) - 3\sin(20t + \frac{\pi}{4})$

$$= 4\left[\frac{1}{2} - \frac{1}{2}\cos(40\pi t)\right] - 3\sin(20t + \frac{\pi}{4})$$

$$= 2 - 2\cos(40\pi t) - 3\sin(20t + \frac{\pi}{4})$$

$\omega_1 = 2\pi f_1 = 40\pi$ $\quad\quad$ $\omega_2 = 2\pi f_2 = 20$

$f_1 = 20$ $\quad\quad$ $f_2 = \frac{10}{\pi}$

$n_1 f_0 = 20$ $\quad\quad$ $n_2 f_0 = \frac{10}{\pi}$

$$\frac{n_1 f_0}{n_2 f_0} = \frac{n_1}{n_2} = \frac{20}{\frac{10}{\pi}}$$ → irrational number ⇒ aperiodic signal

Ex. 6 $\sin\left(\frac{5\pi}{6}t\right) + \cos\left(\frac{3\pi}{4}t\right) + \sin\left(\frac{\pi}{3}t\right)$

$w_1 = 2\pi f_1 = \frac{5\pi}{6}$ $\quad$ $w_2 = 2\pi f_2 = \frac{3\pi}{4}$ $\quad$ $w_3 = 2\pi f_3 = \frac{\pi}{3}$

$f_1 = \frac{5}{12}$ $\quad$ $f_2 = \frac{3}{8}$ $\quad$ $f_3 = \frac{1}{6}$

$GCD\left(\frac{5}{12}, \frac{3}{8}, \frac{1}{6}\right) = \frac{1}{24}$ → rational number periodic signal

$$\begin{array}{c|c} 5 & 5 \\ \hline 1 & 1 \end{array} \qquad \begin{array}{c|c} 3 & 3 \\ \hline 1 & 1 \end{array} \qquad \begin{array}{c|c} 1 & 1 \\ \hline & \end{array}$$

⇒ البسط

1 → 12, $\boxed{24}$, 36, 48

2 → 8, 16, $\boxed{24}$ $\quad$ ⇒ المقام

3 → 6, 12, 18, $\boxed{24}$

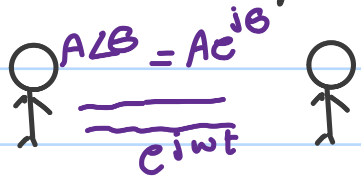
Ex. 7 $x(t) = \cos\left(\frac{10\pi}{3}t\right) + \sin\left(\frac{5\pi t}{4}\right)$

$w_1 = 2\pi f_1 = \frac{10\pi}{3}$ $\quad$ $w_2 = 2\pi f_2 = \frac{5}{4}\pi$

$f_1 = \frac{5}{3}\pi$ $\quad$ $f_2 = \frac{5}{8}\pi$

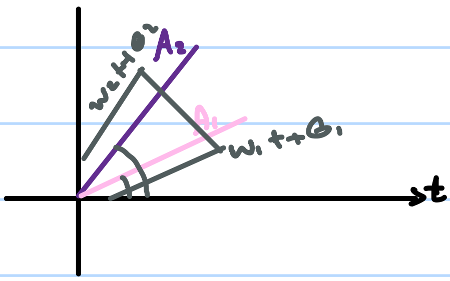
$GCD\left(\frac{5}{3}, \frac{5}{8}\right) = \frac{5}{24}$ → rational number ⇒ Periodic signal

phasor and spectra :-



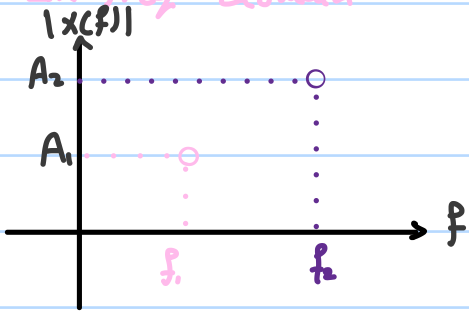
$$x(t) = Re\{ Ae^{j(\omega t+\theta)} \}$$
$$= A\cos(\omega t+\theta)$$

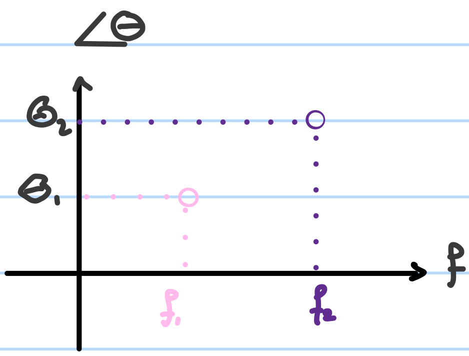
In time domain



$\Rightarrow$

In freq. domain

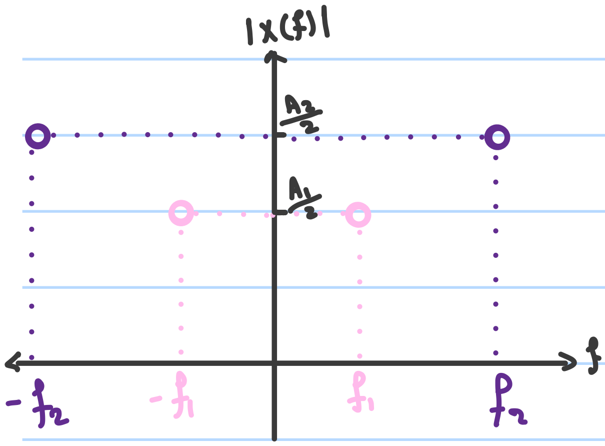


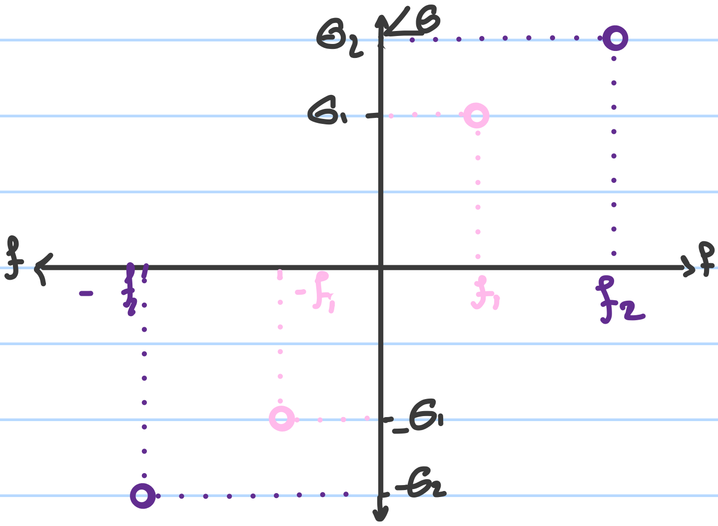


Since :

$$\cos(\omega t+\theta) = \frac{e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}}{2}$$

$$x(t) = \frac{A_1}{2} e^{j(\omega_1 t+\theta_1)} + \frac{A_1}{2} e^{-j(\omega_1 t+\theta_1)} + \frac{A_2}{2} e^{j(\omega_2 t+\theta_2)} + \frac{A_2}{2} e^{-j(\omega_2 t+\theta_2)}$$





Remember :-

$\boxed{\text{even function}}$ $x(t) = \cos(wt) = \cos(-wt)$

Ex. $x(t) = \cos(-3\pi t + \frac{\pi}{4}) = \cos(-(3\pi t - \frac{\pi}{4}))$

$\boxed{\text{odd function}}$ $x(t) = \sin(wt) \Rightarrow \sin(-wt) = -\sin(wt)$

Ex. consider the following signals:

① $x_1(t) = \cos(20\pi t + \frac{\pi}{3}) - \sin(80\pi t + \frac{\pi}{4})$

ⓐ sketch single sided spectra amplitude and phase.

ⓑ Evaluate and plot double sided spectra amplitude and phase.

Ans.

$x(t) = \cos(20\pi t + \frac{\pi}{3}) - \sin(80\pi t + \frac{\pi}{4})$ * $-\sin(\theta) = \cos(\theta + \frac{\pi}{2})$

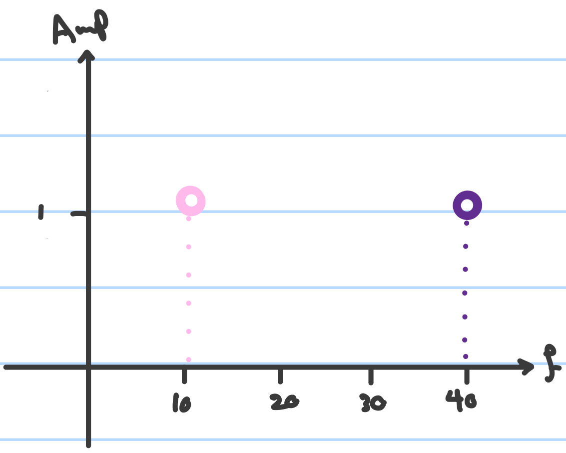
$= \cos(20\pi t + \frac{\pi}{3}) + \cos(80\pi t + \frac{\pi}{4} + \frac{\pi}{2})$

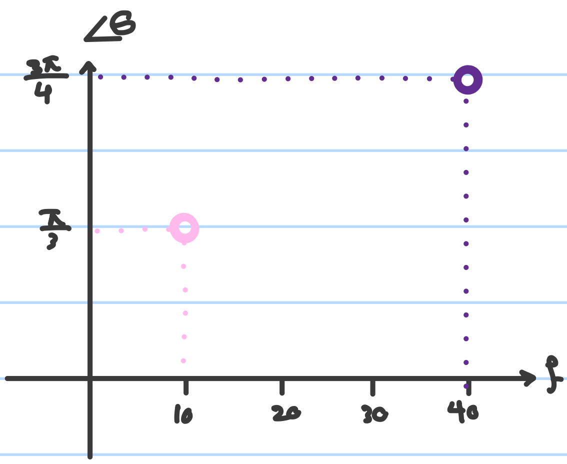
$w_1 = 2\pi f_1 = 20\pi$ $w_2 = 2\pi f_2 = 80\pi$

$f_1 = 10$ $f_2 = 40$

$\theta_1 = \frac{\pi}{3}$ $\theta_2 = \frac{3\pi}{4}$

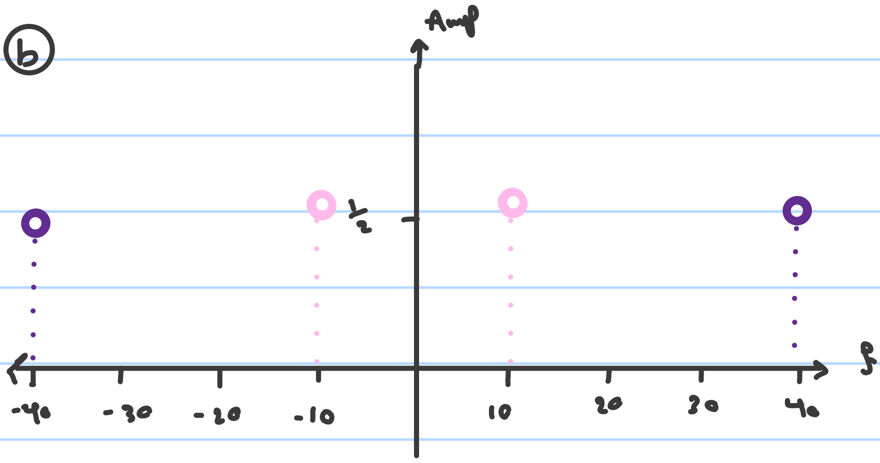
ⓐ

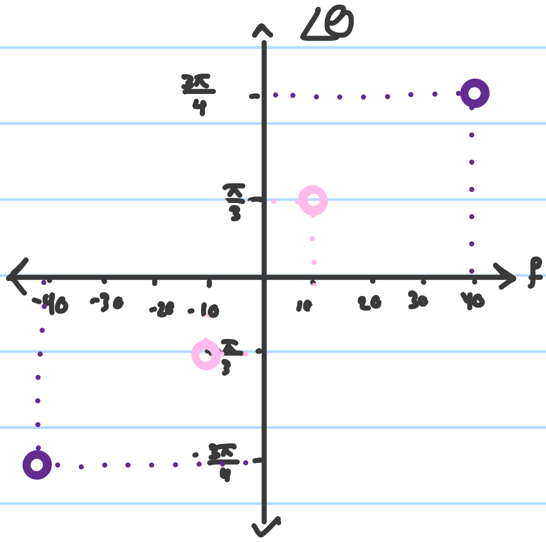




Since: $|X(f)| = |X(-f)|$ even $\angle\theta_X(f) = -\angle\theta_X(-f)$ odd

(b)





(2) $x_2(t) = j2 - \sin^2(20\pi t + \frac{\pi}{5})$

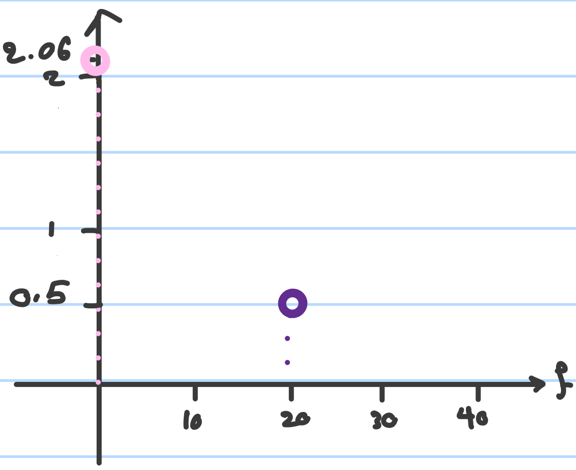
$x_2(t) = j2 - [\frac{1}{2} - \frac{1}{2}\cos(40\pi t + \frac{2\pi}{5})]$

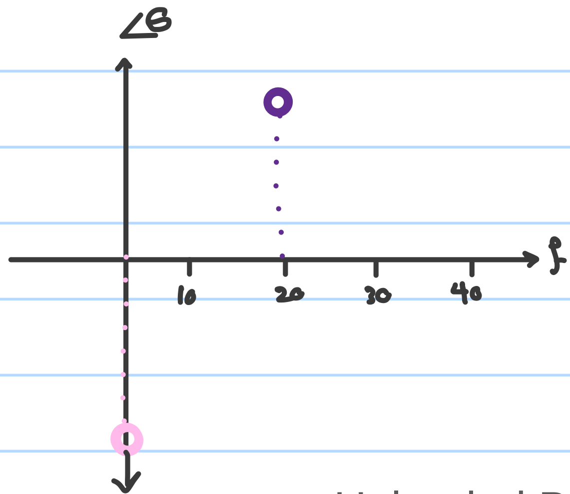
$x(t) = \underbrace{j2 - \frac{1}{2}}_{x\cos(0t+\theta)} + \frac{1}{2}\cos(40\pi t + \frac{2\pi}{5})$

$w_1 = 0$ $w_2 = 2\pi f_2 = 40\pi$

$f_1 = 0$ $f_2 = 20$

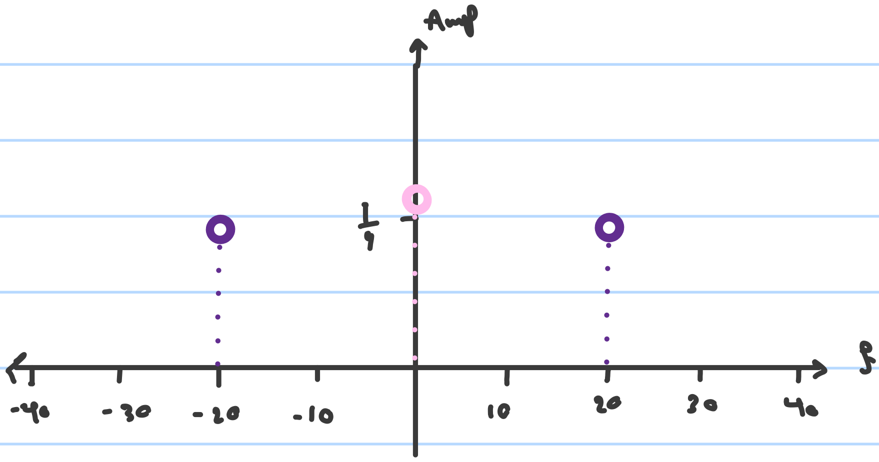
$\sqrt{2^2 + (\frac{1}{2})^2} \angle \tan^{-1}\frac{2}{\frac{1}{2}} \Rightarrow 2.$

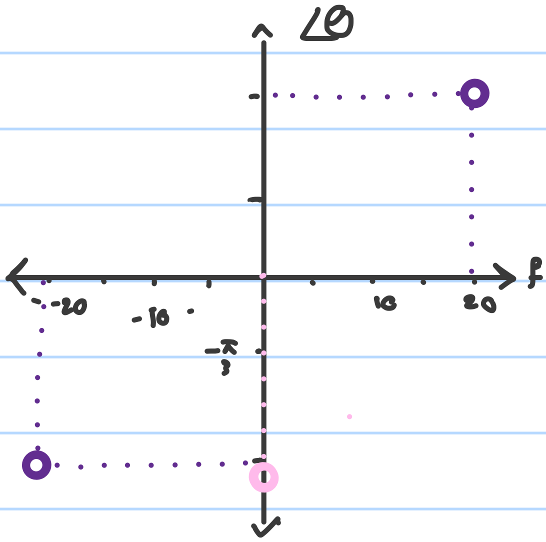




$|x(f)| = |x(-f)| \Rightarrow$ even

$\angle\theta\, x(f) = -\angle\theta\, x(-f)$ odd

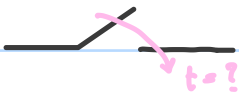




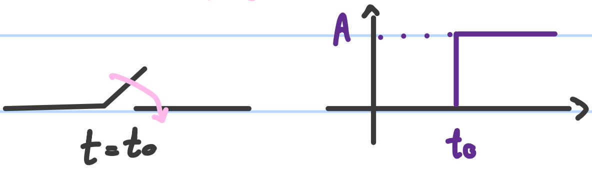
Singularity function:

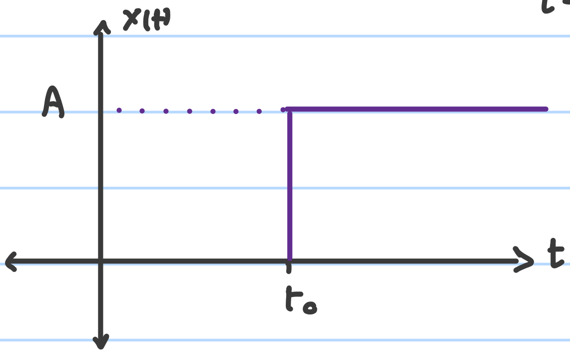
LEC5

1. Step function: $u(t)$



let $x(t) = A\, u(t - t_0)$





$$x(t) \begin{cases} A & t > t_0 \\ 0 & t < t_0 \end{cases}$$

Ex. Sketch the following signals:

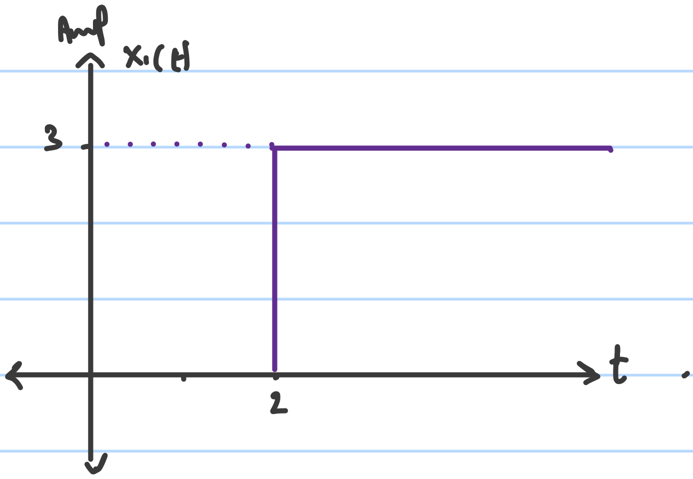
① $x(t) = 3u(2t-4)$

$x(t) = 3u(2(t-2))$

Amp ↗   ← shift

$2t-4 \Rightarrow \boxed{t = 2}$

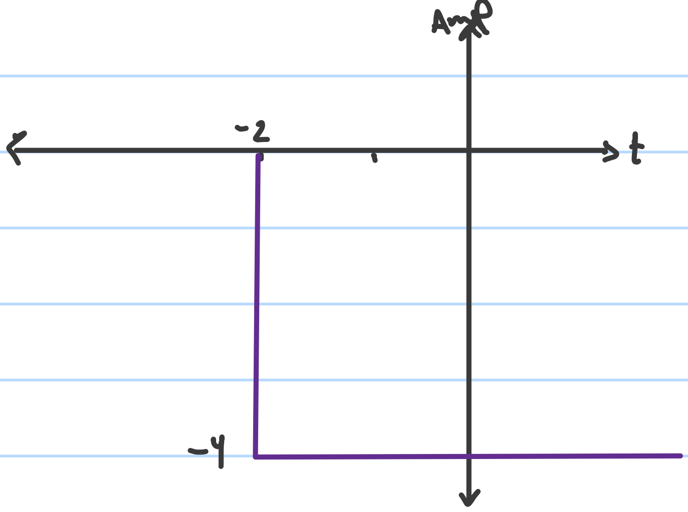
$$x(t) \begin{cases} 3 & t > 2 \\ 0 & t < 2 \end{cases}$$



② $x_2(t) = -4u(2t+4)$

$x(t) = -4\,u(2(t+2))$

$$x(t)\begin{cases} -4 & t > -2 \\ 0 & t < 2 \end{cases}$$



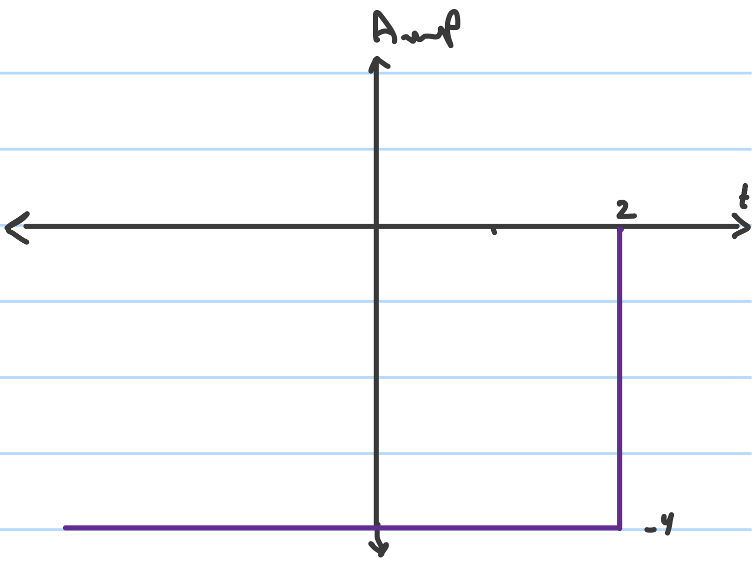
③ $x_3(t) = -4\,u(-3t+6)$

$x_3(t) = -4\,u(-3(t-2))$

انعكاس حول محور السينات

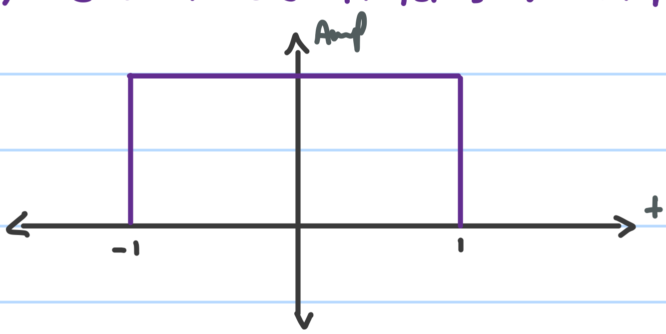
تحديد اذا (-∞) او (∞)

$$x(t) = \begin{cases} -4 & t < 2 \\ 0 & t > 2 \end{cases}$$

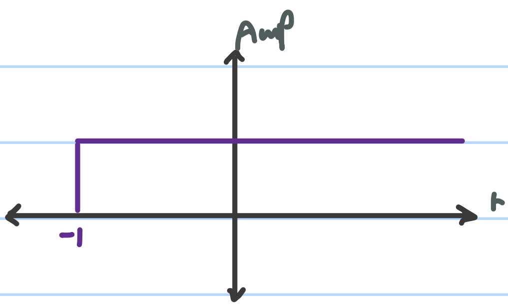


Example: Express $x(t)$ shown below in terms of step functions

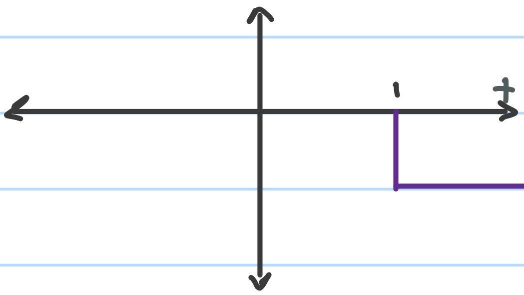
①



option I



$u(t+1)$



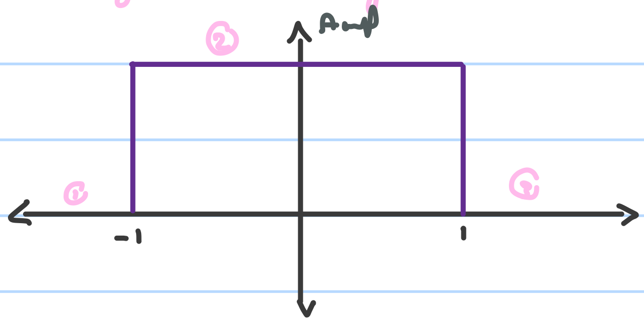
$-u(t-1)$

$$x(t) = u(t+1) - u(t-1)$$

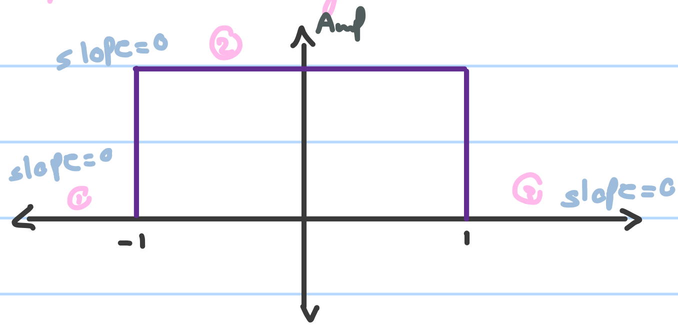
option II $\qquad x(t)=u(t+1)*u(-t+1)$

$\nearrow x$

Ans. ① div. the graph into segments



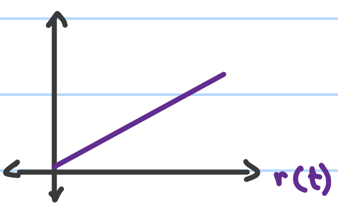
② find slope for each segment



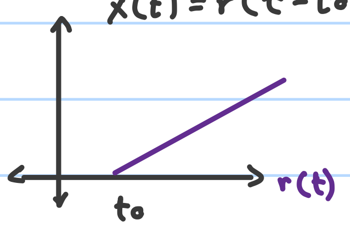
③ r.slope - l.slope => Amp → if slope = 0 --> Amp = Amp(right) - Amp(left)

Ans. $u(t+1) - u(t-1)$

Ramp function:-

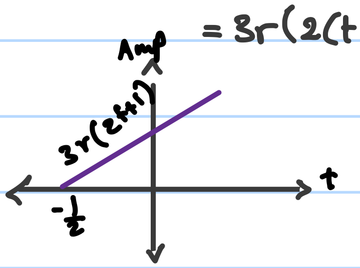


$x(t) = r(t-t_0)$



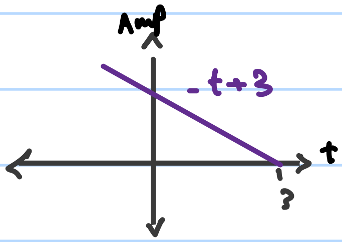
Ex. $x(t) = 3r(2t+1)$

$= 3r\left(2\left(t+\frac{1}{2}\right)\right)$



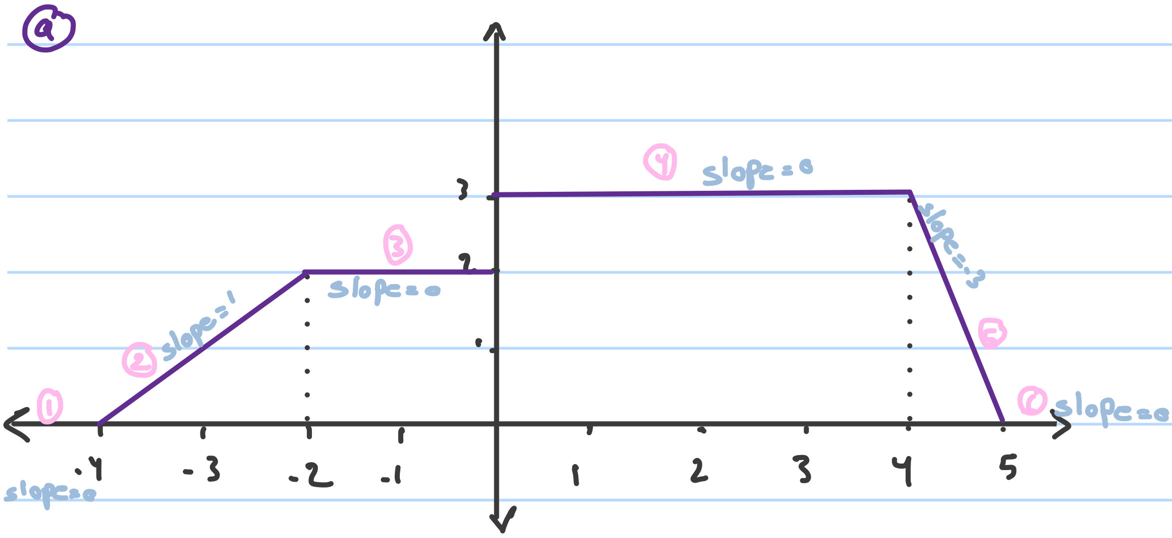
Ex. $x(t) = r(-t+3)$

$x(t) = r(-(t-3))$



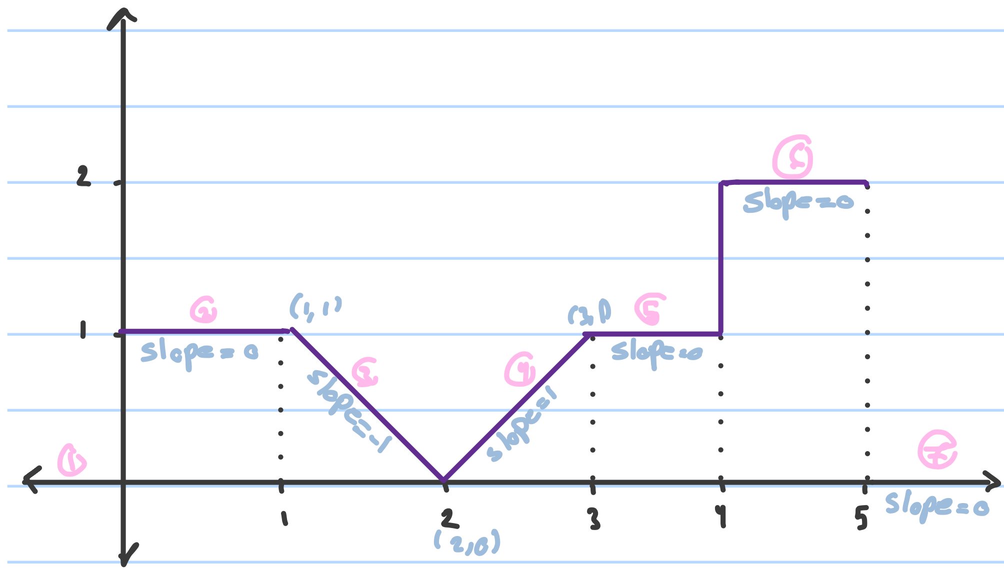
Ex. Express $x(t)$ shown below in terms of step functions

(a)



Ans. $r(t+4) - r(t+2) + u(t) - 3r(t-4) + 3r(t-5)$

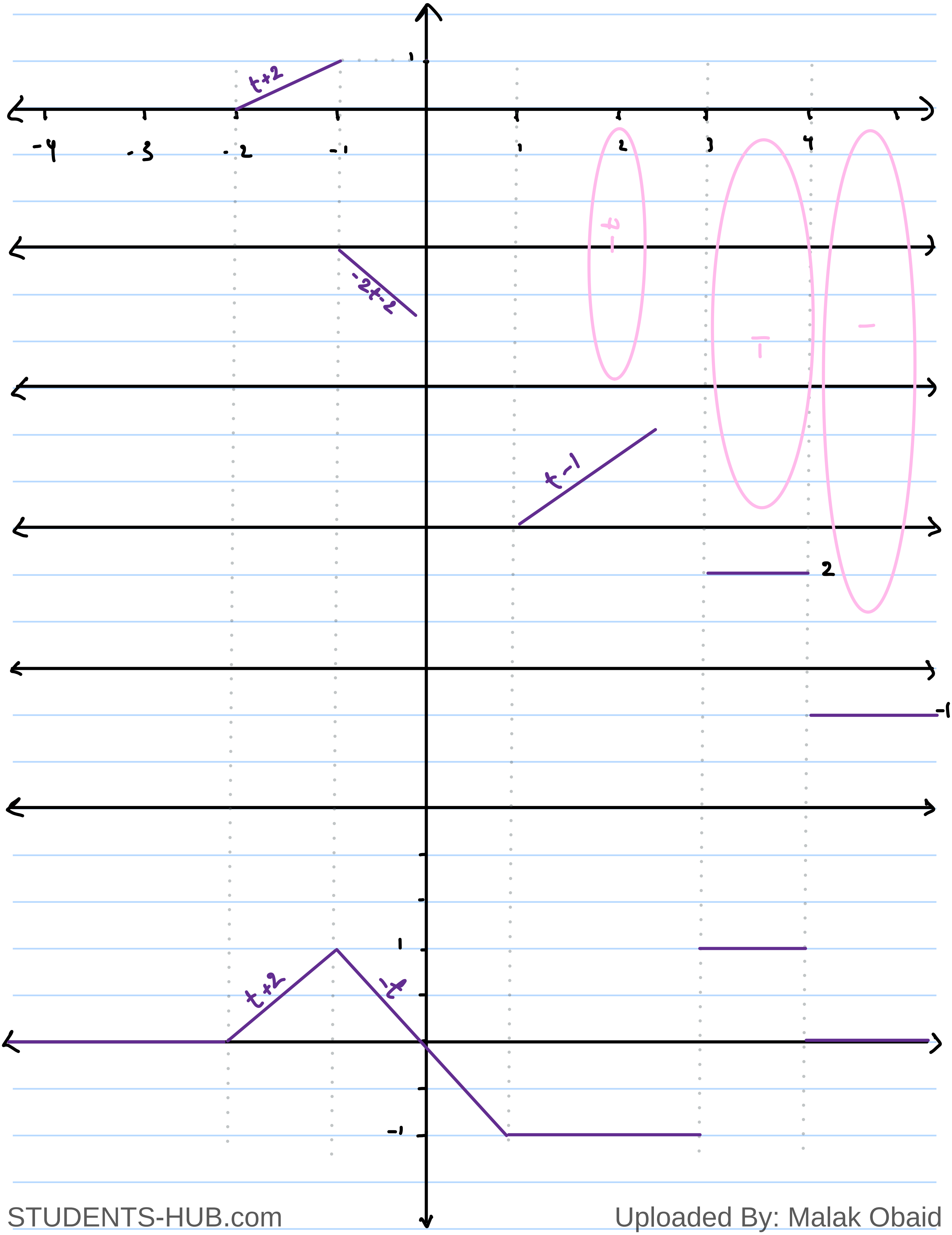
(b)



Ans. $u(t) - r(t-1) + 2r(t-2) - r(t-3) + u(t-4) - 2u(t-5)$

Ex. sketch the following signals:-

① $x(t) = r(t+2) - 2r(t+1) + r(t-1) + 2u(t-3) - u(t-4)$

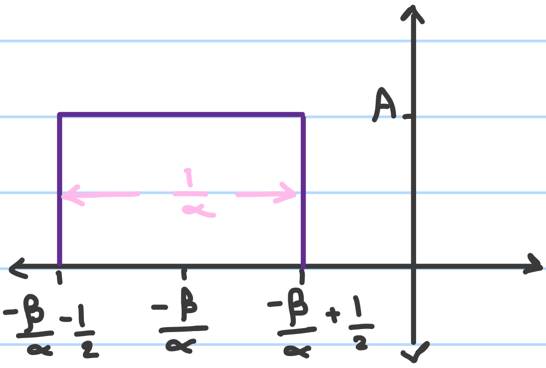


pulse function :- "Π" even function .

In general

$$x(t) = A\Pi(\alpha t + \beta)$$

$$= \underbrace{A}_{\text{Amp}}\Pi\Big(\alpha\big(t + \underbrace{\tfrac{\beta}{\alpha}}_{\text{center}}\big)\Big), \quad \beta \text{ and } \alpha > 0$$

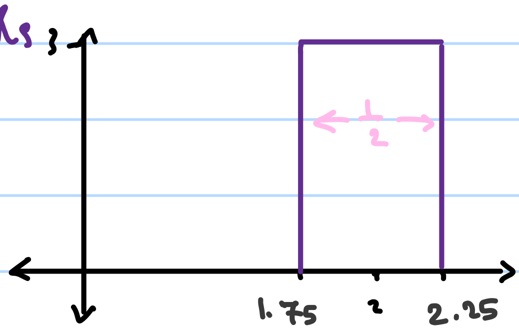


Ex. sketch the following signals :

① $x_1(t) = 3\pi(-2t+4)$

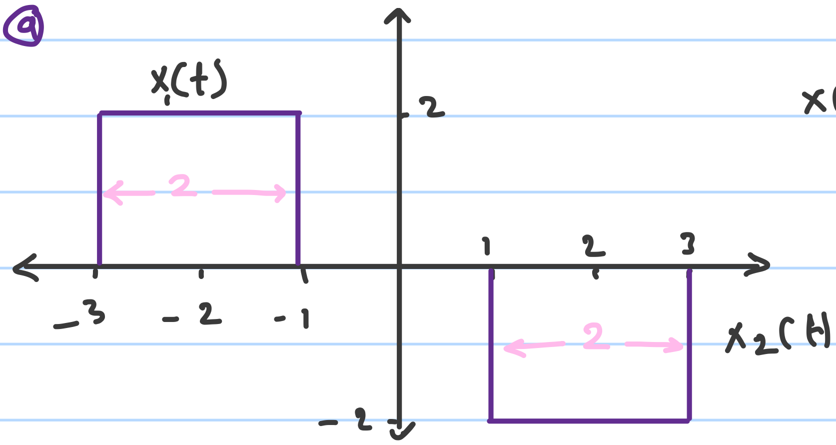
$= 3\pi(\cancel{-}2(t-2))$

$= 3\pi(2(t-2))$



Ex. Express the following signal shown below

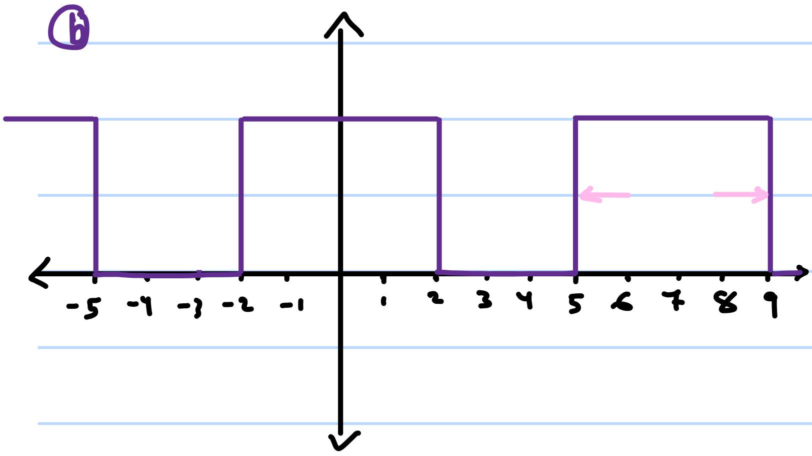
ⓐ



$x(t) = x_1(t) + x_2 t$

$= 2\pi\left(\frac{1}{2}(t+2)\right) - 2\pi\left(\frac{1}{2}(t-2)\right)$

ⓑ

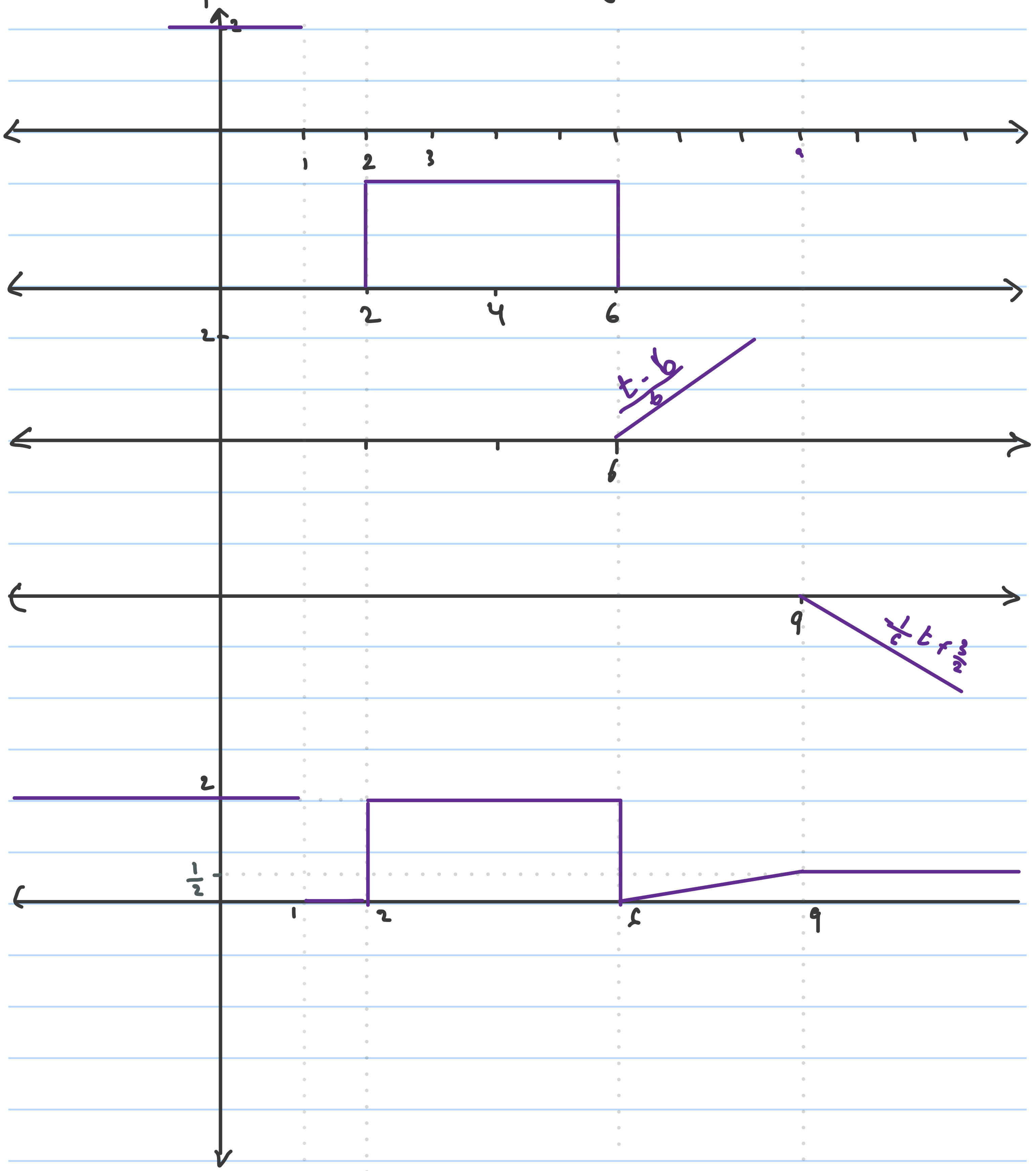


$$= x(t) = \sum_{i=-\infty}^{\infty} \pi\left(\frac{1}{4}(t+7i)\right)$$

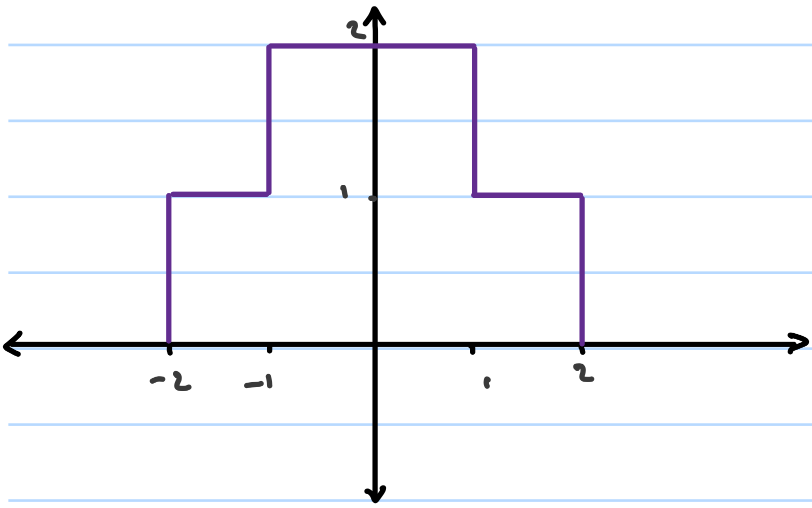
Ex. plot the following signal using elementary signals:

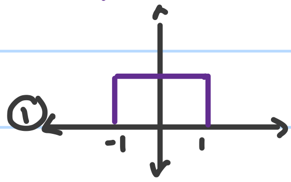
$x(t) = 2\pi\left(\frac{t-4}{4}\right) + r\left(\frac{t-6}{6}\right) - r\left(\frac{t-9}{6}\right) + 2u(1-t)$

$x(t) = 2\pi\left(\frac{1}{4}(t-4)\right) + r\left(\frac{1}{6}(t-6)\right) - r\left(\frac{1}{6}(t-9)\right) + 2u(-(t-1))$

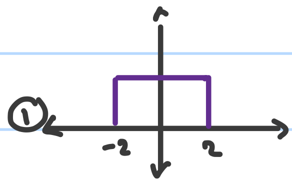


Ex. Express the following signals in terms of puls function LEC7





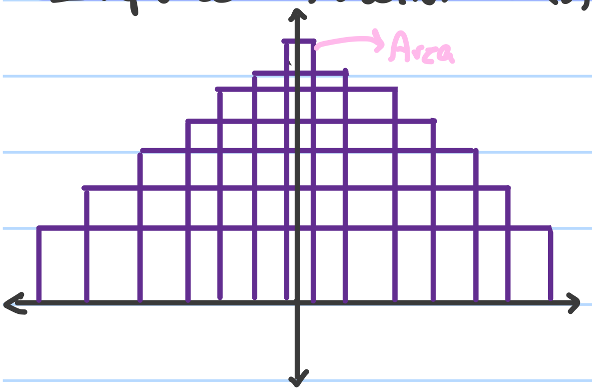
$\pi(\frac{1}{2}t)$

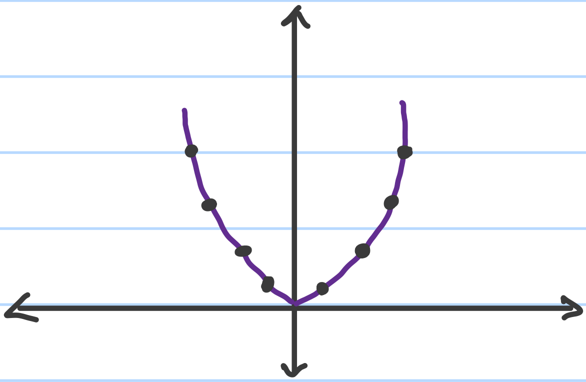


$\pi(\frac{1}{4}t)$

$$x(t) = x_1(t) + x_2(t) = \pi(\tfrac{1}{2}t) + \pi(\tfrac{1}{4}t)$$

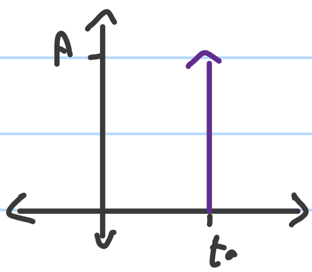
Impulse function $\delta(t)$:-





In general

$$A\delta(t-t_0) = \begin{cases} A & t = t_0 \\ 0 & o.w \end{cases}$$



properties of impulse function

① $\delta(t) = \delta(-t)$ "even function"

② $\delta(at) = \frac{1}{|a|}\delta(t)$ "change of variable"

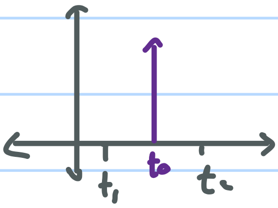
$$\int x(at)\, \partial t$$

$u = at \Rightarrow \partial u = a\, \partial t \Rightarrow \partial t = \frac{1}{a}\partial u \Rightarrow \int x(u)\cdot \frac{1}{a}\, \partial u$

③ $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$ "sampling theorm" $\Rightarrow$ جوابها رقم مضروب ، δ

④ $\int_{t_1}^{t_2} x(t)\,\delta(t-t_0)\,\partial t = \int_{t_1}^{t_2} x(t_0)\,\delta(t-t_0)\,\partial t$ "Sifting theorm"

$$= \begin{cases} x(t_0) & t_1 < t_0 < t_2 \\ 0 & \text{o.w} \end{cases}$$



⑤ $\int_{t_1}^{t_2} x(t)\,\delta^{n}(t-t_0)\,\partial t$ "Derivative theorm" $\Rightarrow$ جوابها رقم

$$= \begin{cases} (-1)^n \left.\dfrac{\partial^n x(t)}{\partial t^n}\right|_{t=t_0} & t_1 < t_0 < t_2 \\ 0 & \text{o.w} \end{cases}$$

⑥ $x(t) * \delta(t-t_0)$?? "Convolution"

Ex. Evaluate the following :-

① $(3t+1)\,\delta(-2t+4)$

$$\underbrace{(3t+1)}_{x(t)}\,\delta(-2(t-2))$$

$$= (3t+1)\cdot\frac{1}{2}\,\delta(t-2) \quad = \frac{7}{2}\,\delta(t-2)$$

② $\displaystyle\int_{3}^{4} (3t+1)\,\delta(-2t+4)\,\partial t$

$$= \int_{3}^{4} (3t+1)\cdot\frac{1}{2}\,\delta(t-2) \quad = 0$$

↳ $\notin [3,4]$

③ $\displaystyle\int_{-2}^{4} (3t+1)\,\delta(-2t+4)\,\partial t$

$$\int_{-2}^{4} 3(t+1)\cdot\frac{1}{2}\,\delta(t-2)$$

$$= 3(2+1)\cdot\frac{1}{2} \quad = \frac{7}{2}$$

④ $\displaystyle\int_{-2}^{4} (3t+1)(2t-1)\,\dot{\delta}(t+1)\,\partial t$

$$(-1)^{1}\,\frac{\partial\,(3t+1)(2t-1)}{dt} = (3t+1)\cdot(2) + (2t-1)\cdot 3$$

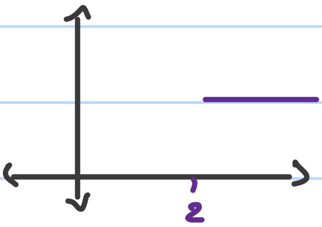
$$= (-1)\cdot\big(\overset{-4}{3(-1)+1}\big)\cdot 2 + \big(\overset{-9}{2\cdot(-1)-1}\big)\cdot 3$$

$$= 13$$

Ex. sketch the following signals

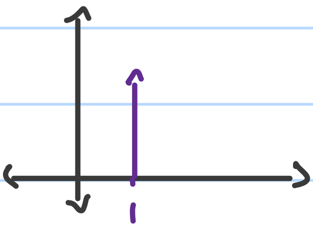
① $\underbrace{u(t-2)}_{x(t)}\,\delta(t-1)$

$= x(1)\,\delta(t-1)$

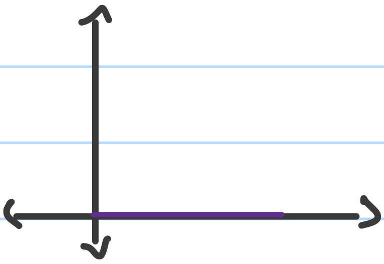


$\Rightarrow u(t-2)$

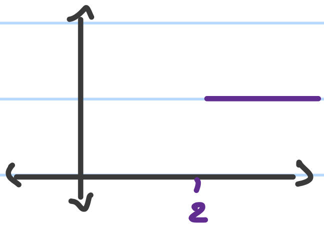
$u(1) = 0$

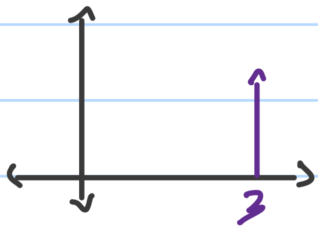


$\Rightarrow$

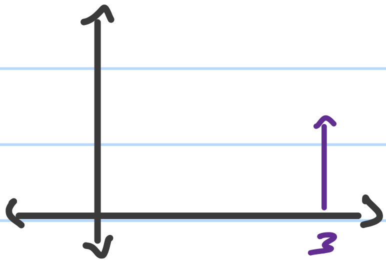


② $u(t-2) \cdot \delta(t-3)$

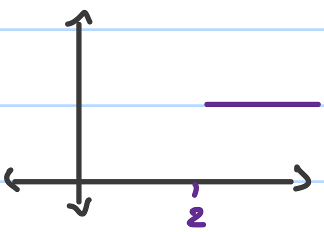




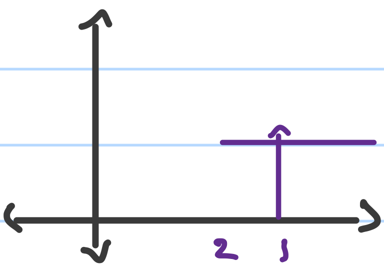
$\Rightarrow$



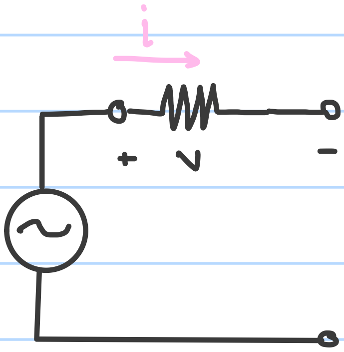
② $u(t-2) + \delta(t-3)$



$\Rightarrow$

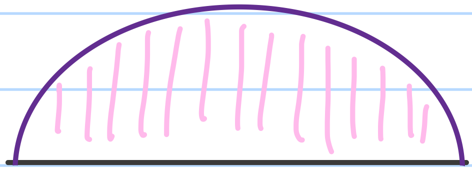


Energy and power signals



$$P_{(t)} = V_{(t)} I_{(t)}$$

$$= i^2 R = \frac{V^2}{R}$$



$$P_{avg} = \lim_{t \to \infty} \frac{1}{2T} \int_{-T}^{T} p(t)\,dt = \lim_{t \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\,dt$$

$$E = \lim_{t \to \infty} \int_{-T}^{T} |x(t)|^2\,dt$$

$P=\lim\limits_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2\,dt$ and $E=\lim\limits_{T\to\infty}\int_{-T}^{T}|x(t)|^2\,dt$

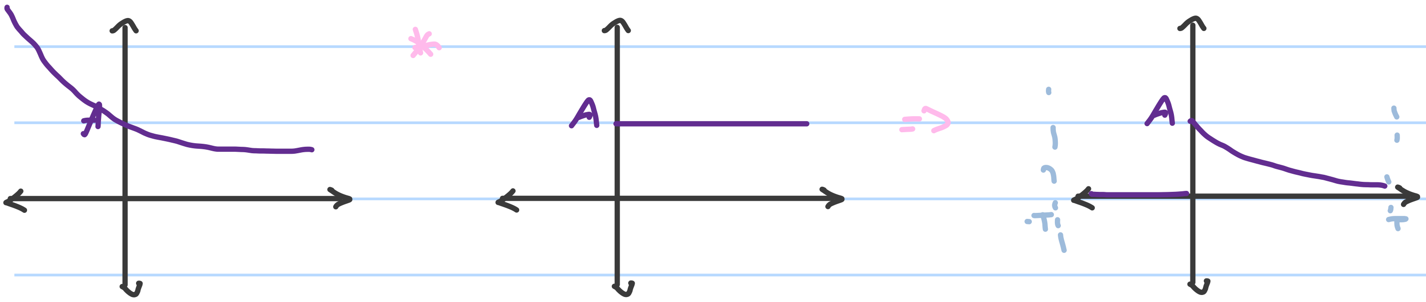
Signal classes

① $x(t)$ is energy signal $0<E\ \infty$ and $P=0$

② $x(t)$ is power signal $0<P<\infty$ and $E=\infty$

Ex. check if the following signals energy or power signals

① $x(t)=Ae^{-\alpha t}u(t)$



$$E=\lim_{T\to\infty}\int_{-T}^{T}|x(t)|^2\,dt=\lim=\lim_{T\to\infty}\cancel{\int_{-T}^{0}(0)^2\,dt}+\int_{0}^{T}(Ae^{-\alpha t})^2\,dt$$

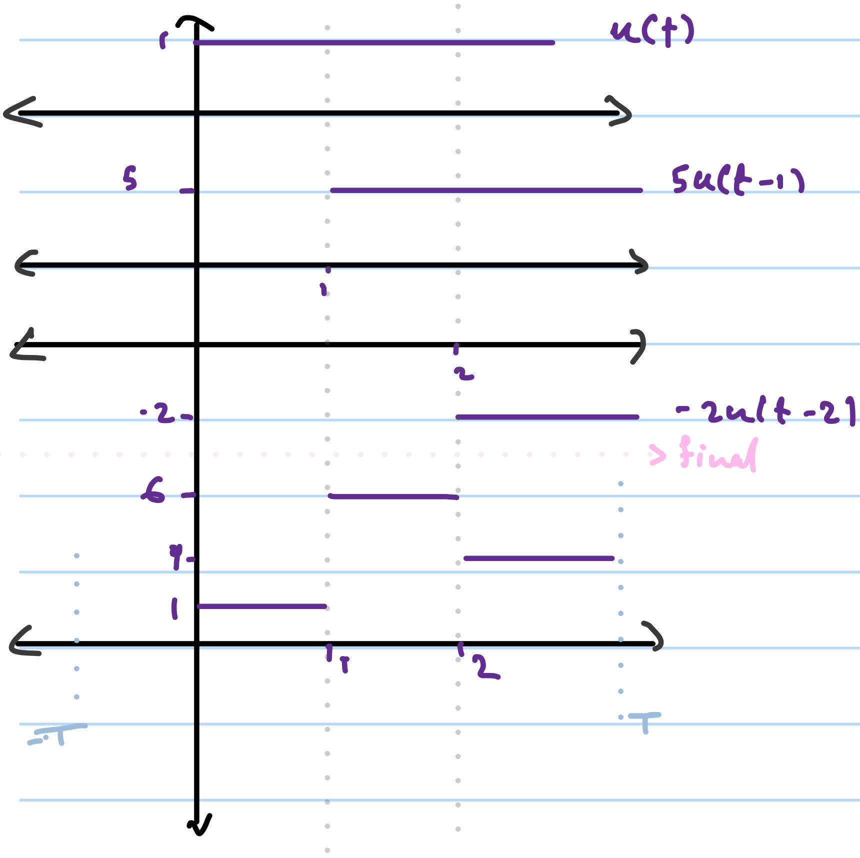
$$=\lim_{T\to\infty}A^2\int_{0}^{T}e^{-2\alpha t}\,dt=\lim_{T\to\infty}\frac{A^2}{-2\alpha}e^{-2\alpha t}\Big|_{0}^{T}$$

$$\lim_{T\to\infty}\frac{A^2}{2\alpha}\left[1-e^{-2\alpha t}\right]=\frac{A^2}{2\alpha}<\infty$$

$$P=\lim\frac{1}{2T}\int_{-T}^{T}|x(t)|^2\,dt=\lim\frac{A^2}{2\alpha}\cdot\frac{1}{2T}=0$$

Energy signal

② $u(t) + 5u(t-1) - 2u(t-2)$



$$E = \lim_{T \to \infty} \int_{-T}^{T} (|x(t)|)^2 \, dt \quad = \lim_{T \to \infty} \int_{-T}^{0} (0)\, dt + \int_{0}^{1} (1)^2 dt + \int_{1}^{2} (6)^2 dt + \int_{2}^{T} (4)^2 dt$$

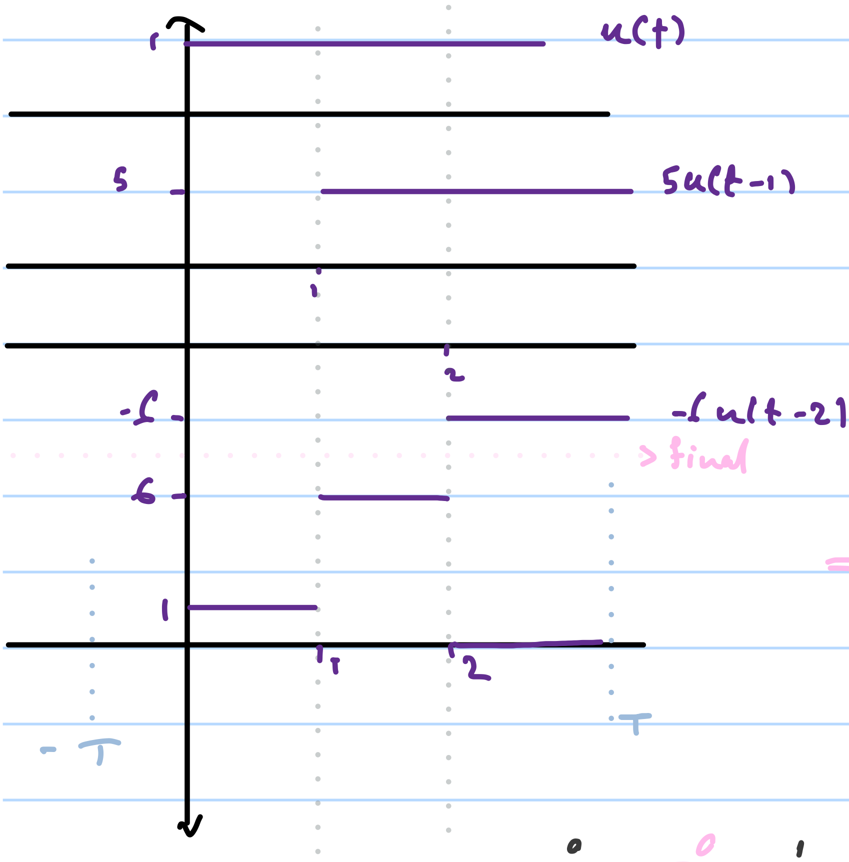
$$= \lim_{T \to \infty} \left[(1)\times(1) + (36)\times(1) + 16\,(T-2) \right] = \infty$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt = \lim \frac{1}{2T} \left[59 + 16\,T \right]$$

$$= 8 < \infty$$

Power signal

③ $x(t) = u(t) + 5u(t-1) - 6u(t-2)$



$\Rightarrow$ bounded or Time limited

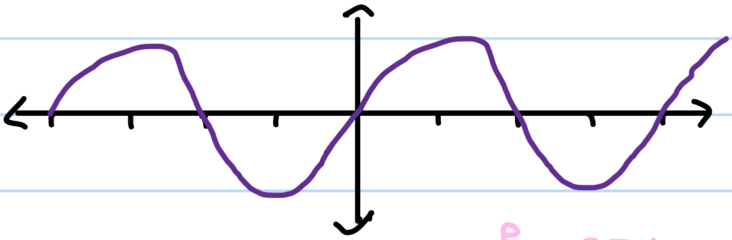
$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \cancel{\int_{-T}^{0} (0)^2 dt} + \int_{0}^{1} (1)^2 dt + \int_{1}^{2} (6)^2 dt + \cancel{\int_{2}^{T} (0)^2 dt}$$

$$= \lim_{T\to\infty} [(1)(1) + (36)(1)] = \boxed{37 < \infty}$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t) dt = \lim_{T\to\infty} \frac{37}{2T} = \boxed{0}$$

Energy Signal

④ $x(t) = A\sin(\omega_0 t)$ → periodic



$\omega = 2\pi f = 2\pi \frac{1}{T_0}$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \int_{-T}^{T} A^2 \sin^2(\omega_0 t)\, dt$$

$$= \lim_{T\to\infty} A^2 \int_{-T}^{T} \left(\frac{1}{2} - \frac{1}{2}\cos 2\omega_0 t\right) dt$$

$$= \lim_{T\to\infty} A^2 \left[\frac{2T}{2} - \frac{1}{2}\cdot\frac{1}{2\omega_0}\left(\cancel{\sin 2\cdot\frac{2\pi}{T}\cdot T}^{\,0} - \cancel{\sin 2\cdot\frac{2\pi}{T}\cdot -T}^{\,0}\right)\right]$$

$$= \lim_{T\to\infty} A^2\left[\frac{2T}{2}\right] = \infty$$

$$P = \frac{1}{2T}\int_{-T}^{T} |x(t)|\, dt = \frac{A^2}{2}$$

Power signal

In general $x(t) = 3\sin(20\pi t) + 4\cos(50\pi t) - 2\sin(80\pi t)$.

↳ periodic signal ⇒ power signal

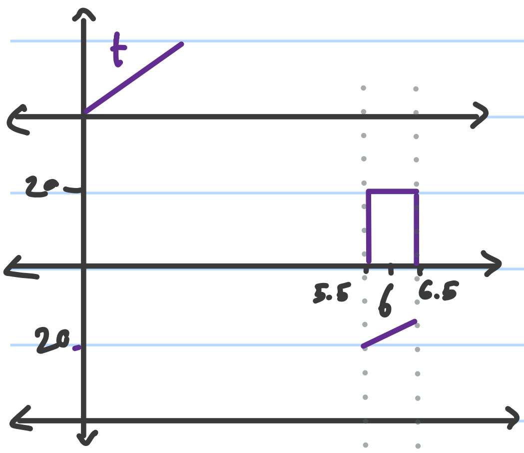
$$P = P_{avg} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 \, dt \equiv \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T} |x(t)|^2 \, dt$$

$$P_{tot} = \overset{4.5}{\frac{3^2}{2}} + \overset{8}{\frac{(4)^2}{2}} + \overset{2}{\frac{(-2)^2}{2}} = 14.5$$

⑤ $y(t) = 20\, r(t)\, \Pi(6-t) + \Pi(0.5t+6)$

$= 20\, r(t)\, \Pi(\cancel{-}(t-6)) + \Pi(0.5(t+12))$

$= 20 r(t)\, \Pi(t-6) + \Pi(\tfrac{1}{2}(t+12))$



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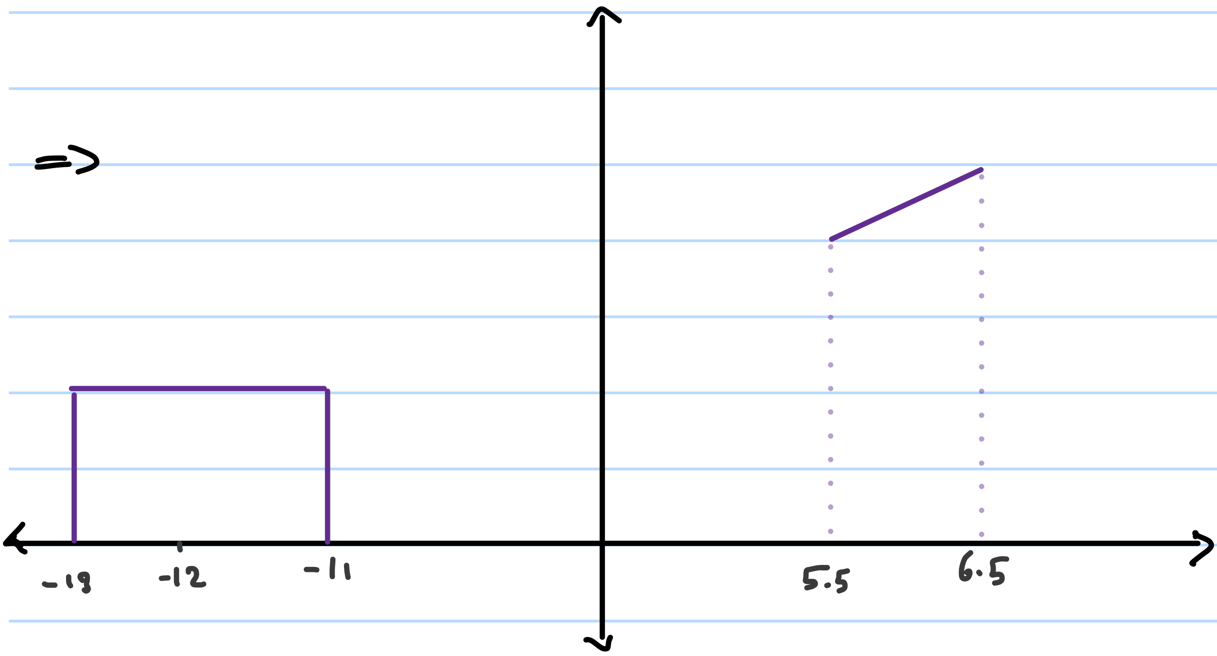


$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|\, dt = \lim_{T\to\infty} \left[\int_{-13}^{-11} (1)^2\, dt + \int_{5.5}^{6.5} 20t\, dt \right]$$

$$= \lim_{T\to\infty} \left[(1)(2) + \frac{20^2 t^2}{2} \Big|_{5.5}^{6.5} \right]$$

$$\lim_{T\to\infty} \left[2 + 200(6.5^2 - 5.5^2) \right]$$

$$= 2402$$

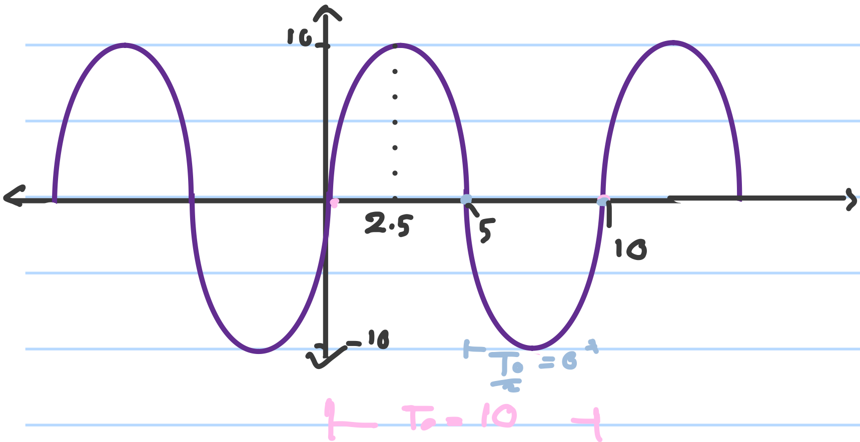
$$P = \lim \frac{1}{2T} \int_{-T}^{T} |x(t)|\, dt = \frac{2402}{2T} = 0$$

Energy Signal

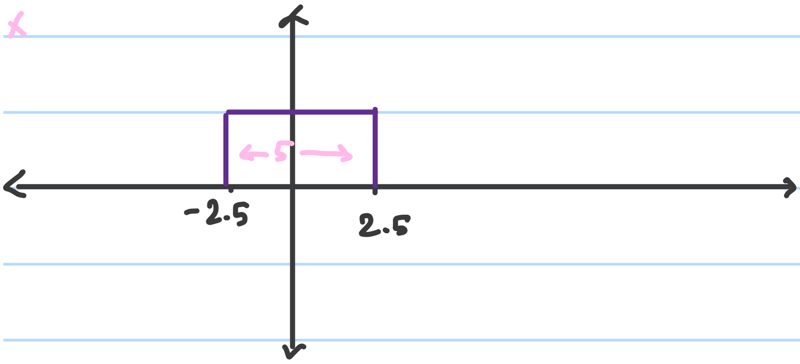
Ex. $x(t) = \underbrace{10\sin(0.2\pi t)}_{x_1(t)} \underbrace{\pi\left(\frac{t}{5}\right)}_{x_2(t)}$

$x_1(t) = 10\sin(0.2\pi t)$

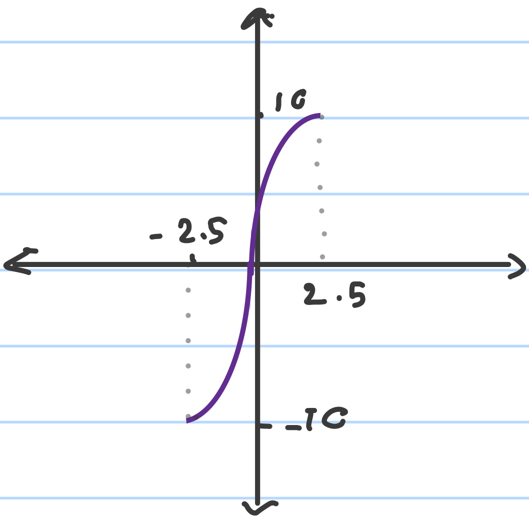
$\omega = 2\pi f = 0.2\pi \Rightarrow f = 0.1\,Hz \Rightarrow T_0 = 10\,sec$



x



$\Rightarrow$



$\Rightarrow$ bounded $\Rightarrow$ Energy signal

$P = 0$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt = \lim_{T\to\infty} \int_{-2.5}^{2.5} 10^2 \sin^2(0.2\pi) \, dt$$

$$= \lim_{T\to\infty} \left[100 \int_{-2.5}^{2.5} \frac{1}{2} - \frac{1}{2} \cos(0.4\pi t] \, dt \right]$$

$$= \lim_{T\to\infty} 100 \cdot \frac{5}{2} - \frac{100}{2} \cdot \left[\sin(0.4\pi . 2.5) - \sin(0.4\pi . -2.5) \right] \to 0$$

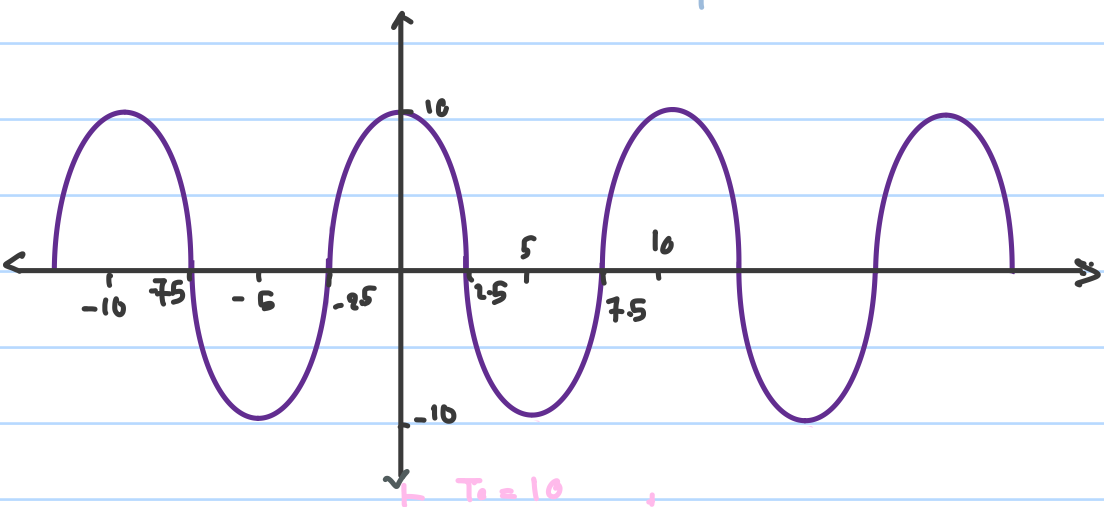
$$= 250 \, J$$

$$P = \lim \frac{1}{2t} \cdot 250 = 0$$

Ex. $\underbrace{\sum_{n=-\infty}^{\infty} 10\cos(0.2\pi t)}_{x_1(t)}\ \underbrace{\pi\left(\frac{t-10n}{5}\right)}_{x_2(t)}$

$x_1(t) = 10\cos(0.2\pi t)$

$w = 2\pi f = 0.2\pi \Rightarrow f = 0.1 \Rightarrow T_0 = \frac{1}{f} = 10\,sec$



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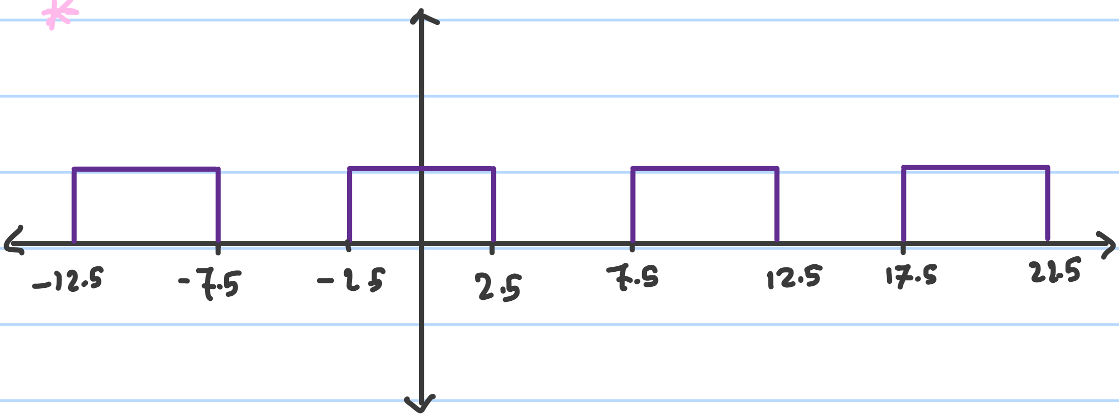
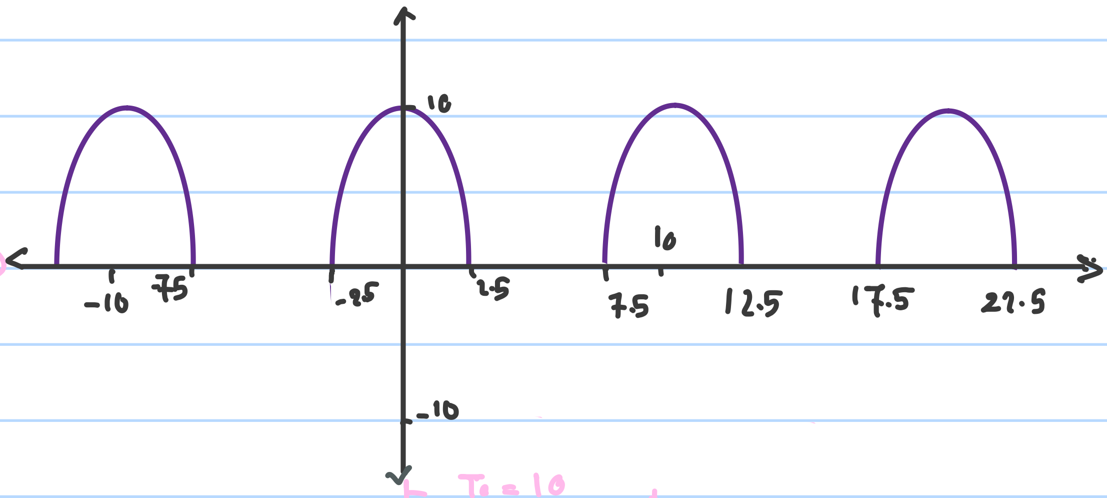


$\Rightarrow$



periodic signal $\Rightarrow$ Power signal $\Rightarrow E = \infty$

$$P = \lim \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \quad \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} 10^2 \cos^2(0.2\pi t) dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left[100 \int_{-T}^{T} \frac{1}{2} + \frac{1}{2} \cos^2(0.2\pi t) dt \right]$$

$$= \lim_{T \to \infty} \frac{1}{2T} \frac{100 \cdot 5(T+T)}{2} + \cancel{\frac{100}{2} \left[\sin(0.2\pi \cdot T) - \sin(0.2\pi \cdot -T) \right.} \to 0$$

$$= \lim_{T \to \infty} \frac{1000\,T}{4T}$$

$$= 250$$

$$E = \infty$$