Torque - Speed characterestics

- At no load $n_{sync} \approx n_m$ (near) → Small slip → Small f_r → small I_R → Small B_R → Small τ_{ind} (E_{ind} is low)
- At heavy load → slip increases → Rotor speed $n_m \downarrow$ → larger I_R → larger f_r → $\delta \uparrow$ But $B_R \uparrow$ → larger τ_{ind}

$\sin\delta$ Keep decreasing until it's almost $= 0$, at this point, increasing load decreases τ_{ind}

This is called **pullout Torque**

Modeling

$$\tau_{ind} = k B_R B_{net} \sin\delta$$

- K is a constant
- $B_R \propto I_R$ → $I_R \propto S$ → $S \frac{1}{\propto} n_m$
- $B_{net} \propto E_1$ → E_1 assumed constant
- $\delta \propto S$ → $S \frac{1}{\propto} n_m$ ($\sin\delta \downarrow$ when $\delta \uparrow$)

Note: $\delta = \theta_R + 90°$

$\sin\delta = \underbrace{\sin(\theta_R + 90°)}_{\cos\theta_R}$

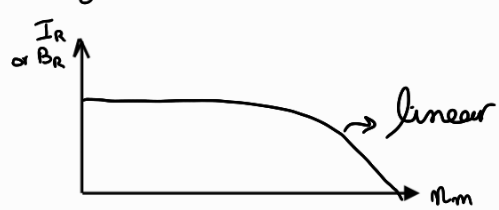
$\cos\theta_R = PF$ of Rotor

$\theta_R = \tan^{-1}(sX_{R_0}/R_R)$

Characterestics curve Regions

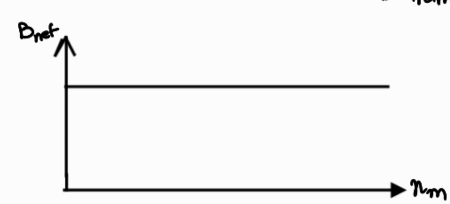
1- low slip Region $0.01 < S < 0.07$

- ▷ In this region, s increases linearly and n_m decreases linearly
- ▷ X_R negligible, $PF \approx 1$
- ▷ I_R increases linearly with s
- ▷ Normal operating range of an induction motor



2- Moderate slip Region

- ▷ PF drops
- ▷ I_R does not increase rapidly with s
- ▷ Peak Torque occurs (pullout torque) → $\tau_{pullout} \approx 200-250\% \, \tau_{rated}$

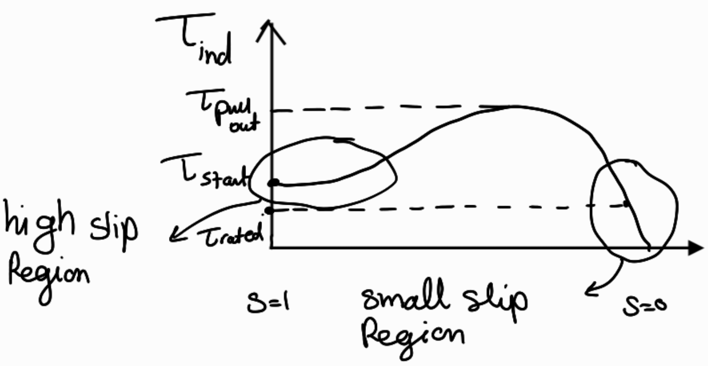


Because $PF = \cos\left(\tan^{-1}\frac{SX_{R_0}}{R_0}\right)$ — These are physical parameters

3- High slip Region

- ▷ $\tau_{ind} \downarrow$ when load $\uparrow$
- ▷ Increase in I_R is totally over shadowed by decrease in PF_R

Starting Torque $\approx 150\% \text{ of } \tau_{rated}$ → So IM can be started at full load



and cannot become zero

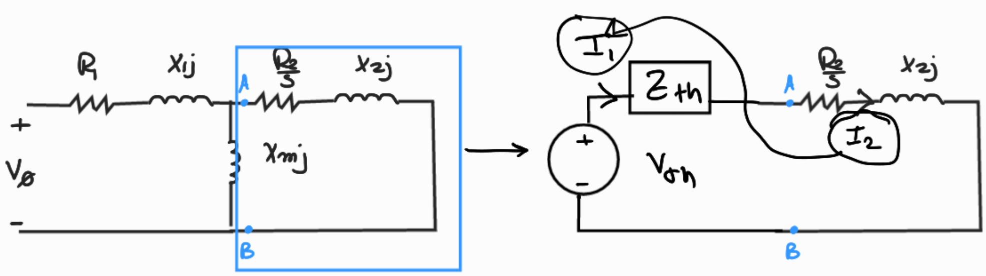
لوّ هنا ال motor قدرة على البدء كل عالي كثير

why is τ_{start} is high Because PF is never zero

Deriving induction Torque equation

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$$

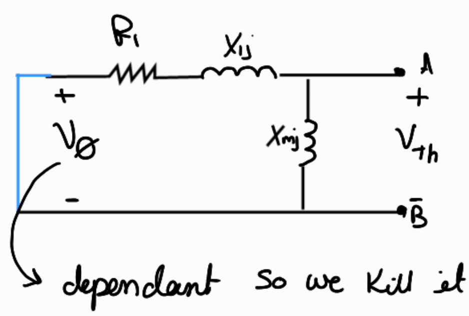
Thevenin equivalent



$V_{Th} = V_{oc}$

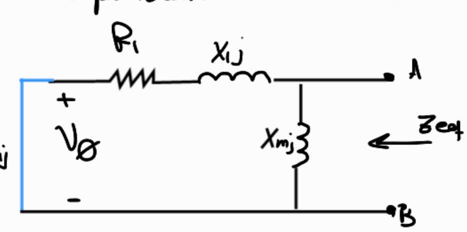
$V_{Th} = \frac{V_\phi X_mj}{R_1 + (R_1 + X_m)j}$ (voltage divider)

$|V_{Th}| = \frac{|V_\phi| \sqrt{X_m^2}}{\sqrt{R_1^2 + (R_1 + X_m)^2}} = \frac{V_\phi X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}}$



$Z_{Th} = Z_{eq} = R_{Th} + X_{Th}j$

where $Z_{eq} = (R_1 + X_1j) // (X_mj) = \frac{(R_1 + X_1j) X_mj}{(R_1 + X_1j) + X_mj}$



If $X_m \gg X_1$, $X_m \gg R_1$

$V_{Th} \simeq \frac{V_\phi X_m}{X_1 + X_m}$

$R_{Th} \simeq R_1 \left(\frac{X_m}{X_1 + X_m}\right)^2$, $X_{Th} \simeq X_1$

$I_1 = \frac{V_{Th}}{Z_{Th} + \frac{R_2}{s} + X_2j}$

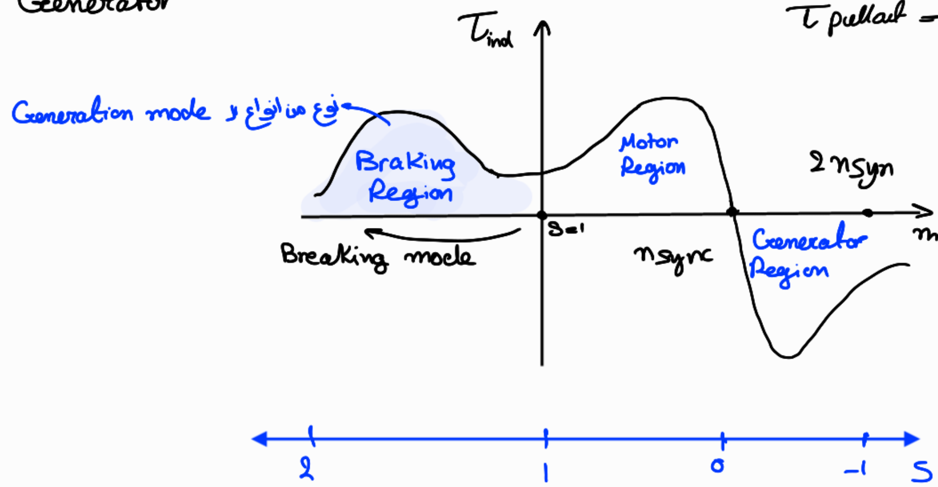
We know that $P_{AG} = 3 I_2 \frac{R_2}{s}$

$\tau_{ind} = \frac{3 I_2^2 R_2/s}{\omega_{sync}} = 3 \times \left(\frac{V_{Th}}{Z_{Th} + \frac{R_2}{s} + X_2j}\right)^2 \times R_2/s \,/\, \omega_{sync}$

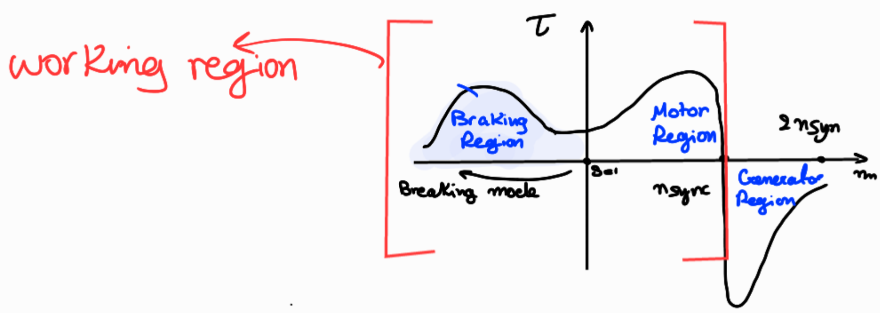
$$\tau_{ind} = \frac{3 \times \left[\frac{V_{Th}}{(R_{Th} + \frac{R_2}{s})^2 + (X_{Th} + X_2)^2} \right]^2 \times \frac{R_2}{s}}{\omega_{sync}}$$

$n > n_{sync} \rightarrow$ Generator

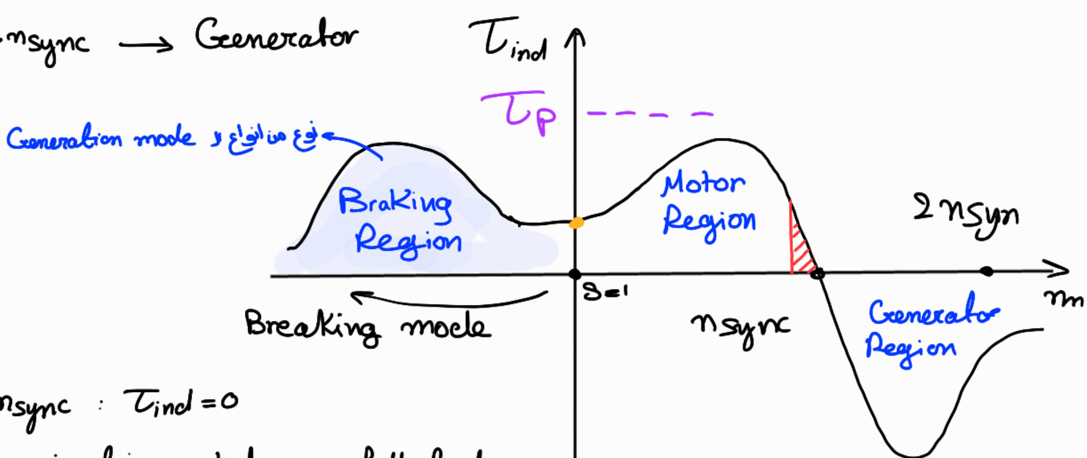
$\tau_{pullout} = \tau_{breakdown}$



For new cars



$n > n_{sync} \rightarrow$ Generator



At n_{sync} : $\tau_{ind} = 0$

* curve is linear between full load and no load

* $\tau_{max} = \tau_{pullout} = 2\text{–}3\ \tau_{rated}$

* Starting $\tau = 1.5\ \tau_{rated}$

τ_{ind} depends on : $(\text{Voltage app})^2$

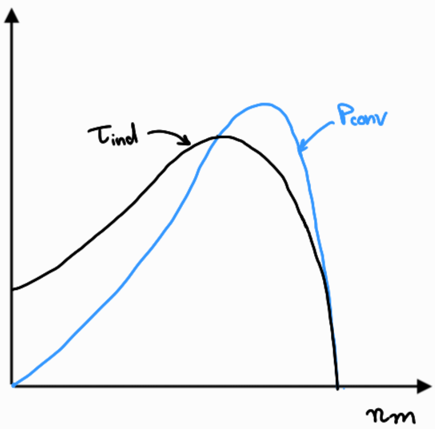
n_{syn}

How does Braking occur?

when the magnetic field rotation is reversed by switching two stator phases the motor will be turning backward relative to it, τ_{ind} will stop the machine rapidly (braking) → This is called plugging (plug reversal)

Converted power

- $P_{conv} = \tau_{ind}\,\omega_m$
- P_{max} occus at different speed to τ_{max}

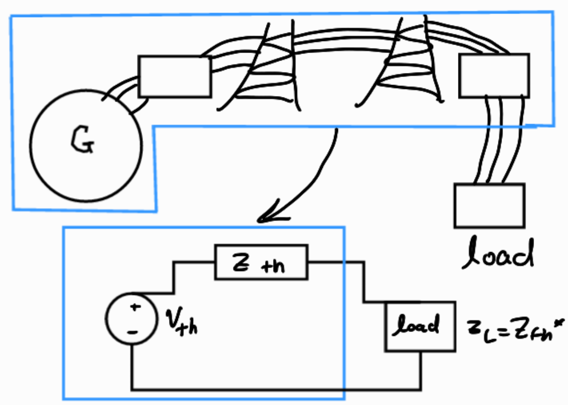


Maximum Pullout Torque

Pullout torque when power consumed by $\frac{R_2}{S}$ is highest

$$\frac{R_2}{S} = \sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}$$

$$s_{max} = \frac{R_2}{\sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}}$$



$$P_{L\,max} = \frac{V_{th}^2}{8R_L}$$

$$\tau_{max} = \frac{3V_{Th}^2}{2\omega_{sync}\left[R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}\right]}$$

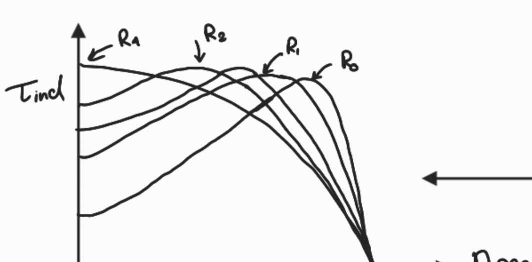
τ_{max}:

- Torque is related to the square of the applied voltage
- Torque is also inversly proportional to the machine impedances
- Slip is dependent upon rotor resistance

Torque is independant to rotor resistance

Adding resistance to machine impedances:

- starting torque
- Max pullout speed



- τ_{max} is independant of R_2
- As R_2 increases S_{max} increases
- Max torque remains constant but pullout speed decreases
- Starting torque increases

Speed Control of induction motors

1- pole changing

2- Stator Voltage control

3- Supply Frequency control

4- Rotor resistance control

Note:

- changing Frequency will change reactances and V_{rated}

reducing f → increases by factor f_{new}/f_{old} → $X_{new} = \frac{f_{new}}{f_{old}} X_{old}$

increasing f → decreases by factor f_{new}/f_{old} ← same

$V_{rated} = \frac{f_{new}}{f_{old}} V_{old}$