

4.2 The Optimal values of a Linear Function Subject to Constraints

For a **closed** and **bounded** region, we can find the optimal (maximum or minimum) value of the objective function by evaluating the function at each of the corners of the solution region.

Example:- Find the maximum and minimum value of $C = 2x + y$ subject to the constraints

$$\begin{cases} x + 2y \leq 10 \\ 2x + y \leq 14 \\ x \geq 0, y \geq 0 \end{cases}$$

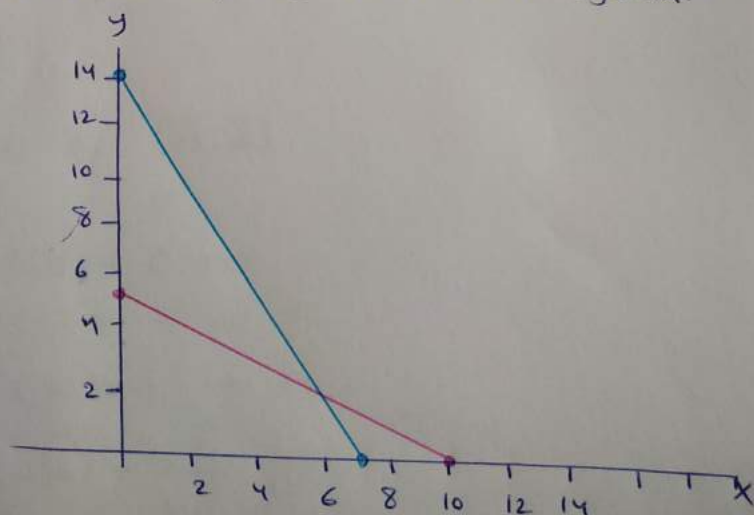
① Graph the solution of the constraint system:-

$$x + 2y = 10$$

x	y
0	5
10	0

$$2x + y = 14$$

x	y
0	14
7	0



$x \geq 0, y \geq 0 \rightarrow$ Quadrant I

For $x + 2y \leq 10$

take $(0,0)$ as a test point

$$0 + 2(0) \stackrel{??}{\leq} 10$$

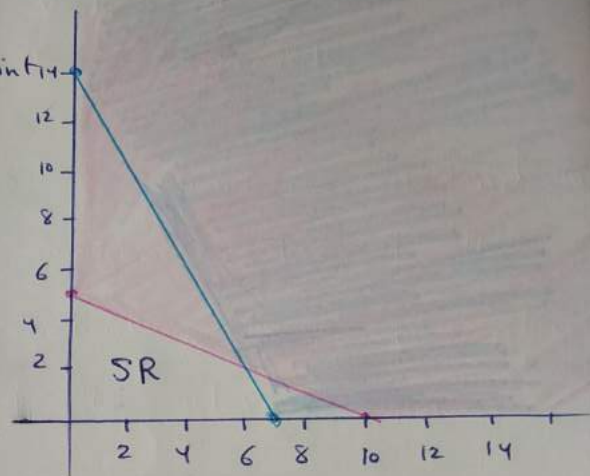
$$0 \leq 10 \checkmark$$

For $2x + y \leq 14$

take $(0,0)$ as a test point

$$2(0) + 0 \stackrel{??}{\leq} 14$$

$$0 \leq 14 \checkmark$$



② Corners : $(0, 5)$, $(0, 0)$, $(7, 0)$

$$2x + y = 14$$

$$(x + 2y = 10) \times -2$$

$\rightarrow$

$$2x + y = 14$$

$$-2x - 4y = -20$$

$$\rightarrow -3y = -6$$

$$\rightarrow \boxed{y = 2}$$

$$2x + y = 14 \rightarrow 2x + 2 = 14 \rightarrow 2x = 12$$

$$\rightarrow \boxed{x = 6}$$

The fourth corner is $(6, 2)$

③ At $(0, 0)$: $C = 2(0) + 0 = 0$

At $(0, 5)$: $C = 2(0) + 5 = 5$

At $(7, 0)$: $C = 2(7) + 0 = 14$

At $(6, 2)$: $C = 2(6) + 2 = 18$

$C = 18$ is the maximum value at $x = 6, y = 2$

$C = 0$ is the minimum value at $x = 0, y = 0$

Example:- Use the given feasible region determined by the constraint inequalities to find the maximum and minimum of the objective function $f = 2x + 7y$ (if they exist)

$$\begin{aligned} \text{At } (2, 2) :- f &= 2(2) + 7(2) \\ f &= 4 + 14 = 18 \end{aligned}$$

$$\begin{aligned} \text{At } (6, 4) :- f &= 2(6) + 7(4) \\ f &= 12 + 28 = 40 \end{aligned}$$

$$\text{At } (6, 1) :- f = 2(6) + 7(1) = 12 + 7 = 19$$

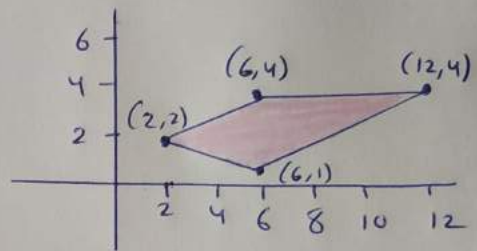
$$\text{At } (12, 4) :- f = 2(12) + 7(4) = 24 + 28 = 52$$

f has a maximum value at $x = 12$ and $y = 4$

f has a minimum value at $x = 2$ and $y = 2$

The maximum value of f is 52

The minimum value of f is 18



Example:- The graph of the feasible region is shown.

Find the corners of ~~each~~ the feasible region, and then find the maximum and minimum of the objective function $g = 12x + 8y$

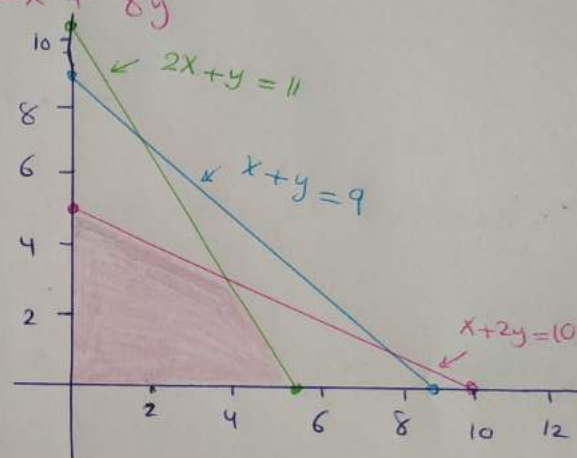
Corners:

at $x + 2y = 10$

y-intercept: put $x = 0$

$$0 + 2y = 10 \rightarrow (y = 5)$$

The first corner is $(0, 5)$



on $2x + y = 11$

x-intercept: put $y = 0$

$$2x + 0 = 11 \rightarrow x = \frac{11}{2} = 5.5$$

The second corner is $(5.5, 0)$

The third corner on $x + 2y = 10$ and $2x + y = 11$

$$\begin{array}{rcl} x + 2y = 10 & \rightarrow & x + 2y = 10 \rightarrow -3x = -12 \rightarrow x = 4 \\ \times 2 (2x + y = 11) & \rightarrow & -4x - 2y = -22 \end{array}$$

$$x + 2y = 10 \rightarrow 4 + 2y = 10 \rightarrow y = \frac{6}{2} = 3$$

The third corner is $(4, 3)$

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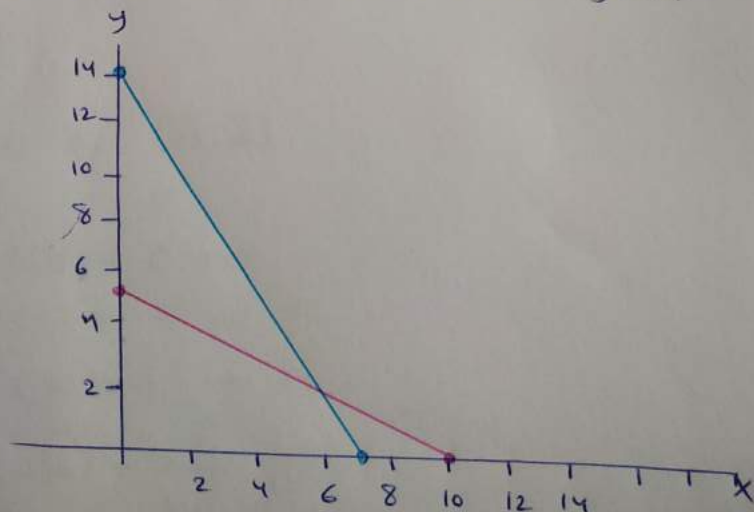
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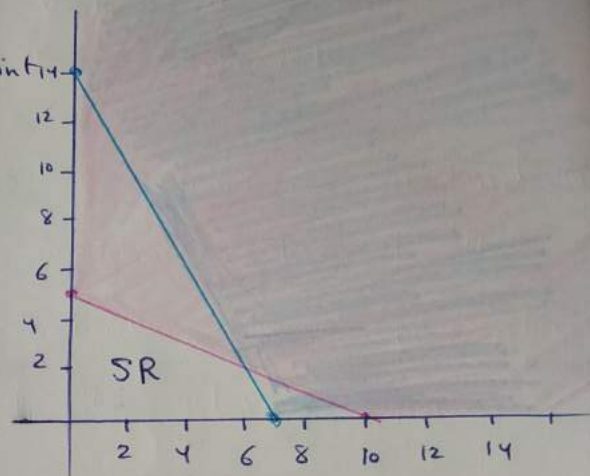
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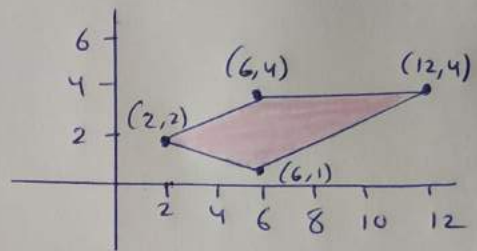
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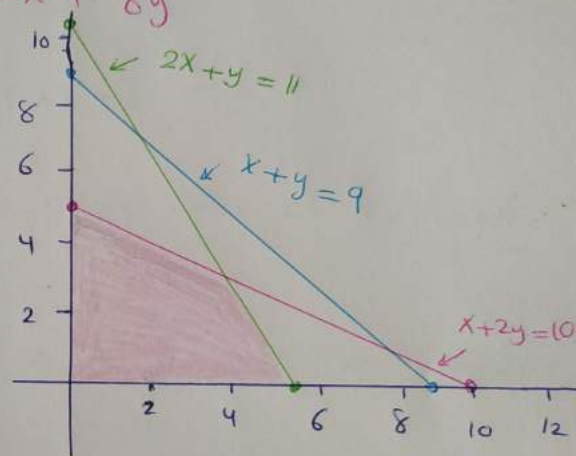
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$$x + 2y = 10 \rightarrow 4 + 2y = 10 \rightarrow y = \frac{6}{2} = 3$$

The third corner is $(4, 3)$



At $(0, 5)$:-

$$g = 12(0) + 8(5) = 40$$

At $(5.5, 0)$:-

$$g = 12(5.5) + 8(0) = 66$$

At $(4, 3)$:-

$$g = 12(4) + 8(3) = 48 + 24 = 72$$

The maximum value of g is 72 at $x=4$ and $y=3$

The minimum value of g is 40 at $x=0$ and $y=5$