Leen Sabri

Q10 $\frac{x}{\sqrt[3]{1+x}} = x(1+x)^{-1/3} = x\left(1+\sum\binom{-\frac{1}{3}}{k}x^k\right)$ $m=\frac{-1}{3}$

$= x\left(1+\binom{-\frac{1}{3}}{1}+\binom{-\frac{1}{3}}{2}x^2+\binom{-\frac{1}{3}}{3}x^3+\dots\right.$

$= x\left(1+\frac{-1}{3}x+\frac{-\frac{1}{3}(\frac{-1}{3}-1)x^2}{2}+\frac{-\frac{1}{3}(\frac{-1}{3}-1)(\frac{-1}{3}-2)x^3}{3!}\right.$

$= x\left(1-\left(\frac{-1}{3}\right)x+\frac{(-\frac{1}{3})(-\frac{4}{3})x^2}{2!}+\frac{(-\frac{1}{3})(-\frac{4}{3})(-\frac{7}{3})x^3}{3!}+\dots\right)$

$= x-\frac{1}{3}x^2+\frac{2}{9}x^3-\frac{14}{81}x^4+\dots$

Q26 $F(x)=\int_0^x t^2e^{-t^2}\,dt$, $[0,1]$

Maclurian:

$e^t = 1+t+\frac{t^2}{2!}+\frac{t^3}{3!}+\dots$

$e^{-t} = 1+(-t)+\frac{(-t)^2}{2!}+\frac{(-t)^3}{3!}+\dots$

$= 1-t+\frac{t^2}{2!}-\frac{t^3}{3!}+\dots$

$e^{-t^2} = 1-t^2+\frac{t^4}{2!}-\frac{t^6}{3!}+\dots$

$= \int_0^x t^2\left[1-t^2+\frac{t^4}{2!}-\frac{t^6}{3!}+\dots\right]dt$

$= \int_0^x \left[t^2-t^4+\frac{t^6}{2!}-\frac{t^8}{3!}+\dots\right]dt$

$= \left[\frac{t^3}{3}-\frac{t^5}{5}+\frac{t^7}{7(2!)}-\frac{t^9}{9(3!)}+\dots\right]_0^x$

$= \frac{x^3}{3}-\frac{x^5}{5}+\frac{x^7}{7(2!)}-\frac{x^9}{9(3!)}+\dots$

نجرب ناخذ لعند الحد الأول

عوضت x=1 لأنه في الفترة [0,1]

$\int_0^x t^2e^{-t^2}\,dt \approx \frac{x^3}{3} = \frac{1}{3}$ with error $< \left|\frac{x^5}{5}\right| = \frac{1}{5} = 0.2$

إذاً ما بزبط لأنه $0.01 = 10^{3} < 0.2$ بنجرب الحد الثاني

$\int_0^x t^2e^{-t^2}\,dt \approx \frac{x^3}{3}-\frac{x^5}{5} = \frac{1}{3}-\frac{1}{5} \approx 0.1333$ with error $\left|\frac{x^7}{7(2!)}\right| = \frac{1}{14} = 0.07$

برضو ما بزبط لانها اكبر من 0.01 نجرب الحد الثالث

$\int_0^x t^2e^{-t^2}\,dt \approx \frac{x^3}{3}-\frac{x^5}{5}+\frac{x^7}{7(2!)}$ with error $< \left|\frac{x^9}{9(3!)}\right| = \frac{1}{54} \approx 0.02$

بنجرب الحد الرابع

$\int_0^x t^2e^{-t^2}\,dt \approx \frac{x^3}{3}-\frac{x^5}{5}+\frac{x^7}{7(2!)}-\frac{x^9}{9(3!)}$ with error $< \left|\frac{x^{11}}{11(4!)}\right| = \frac{1}{264} \approx 3.787\times10^{-3}$

بنجرب الحد الخامس

$\int_0^x t^2e^{-t^2}\,dt \approx \frac{x^3}{3}-\frac{x^5}{5}+\frac{x^7}{7(2!)}-\frac{x^9}{9(3!)}+\frac{x^{11}}{11(4!)}$ with error $< \left|\frac{x^{13}}{13(5!)}\right| = \frac{1}{1560} = 6.41\times10^{-4}$

هلا زبط لأنه $10^{-3} > 6.41\times10^{-4}$

Hence $P_{13}(x) = \frac{x^3}{3}-\frac{x^5}{5}+\frac{x^7}{7(2!)}-\frac{x^9}{9(3!)}+\frac{x^{11}}{11(4!)} \approx F(x)$

Q30 $\lim_{x\to0}\frac{e^x-e^{-x}}{x}$

$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots$

$e^{-x} = 1+x+\frac{(x)^2}{2!}+\frac{(-x)^3}{3!}+\dots$

$= 1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\dots$

$= \lim_{x\to0}\frac{1}{x}\left[\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots\right)-\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\dots\right)\right]$

$= \lim_{x\to0}\frac{1}{x}\left[\cancel{1}+x+\cancel{\frac{x^2}{2!}}+\frac{x^3}{3!}+\dots\cancel{-1}+x-\cancel{\frac{x^2}{2!}}+\frac{x^3}{3!}+\dots\right]$

$= \lim_{x\to0}\frac{1}{x}\left[2x+\frac{2x^3}{3!}+\frac{2x^5}{5!}+\dots\right]$

$= \lim_{x\to0}\left[2+\frac{2x^2}{3!}+\frac{2x^4}{5!}+\dots\right]$

$= 2$