Chapter 7.1, Problem 13E

Problem

Let *J*5 = {0, 1, 2, 3, 4}, and define functions *f: J*5→*J*5 and *g : J*5→ *J*5 as follows: For each *x* ∈ *J*5,

f (*x*) = (*x* + 4)2 *mod* 5 and *g*(*x*) = (*x*2 + 3*x* + 1) *mod* 5.

Is *f* = *g*? Explain.

Step-by-step solution

Step 1 of 2

Consider the function $f : J_5 \to J_5$ and $g : J_5 \to J_5$ for all $x \in J_5$.

Function $f(x) \text{and} \, g(x)$ is defined as:

$$f(x) = (x+4)^2 \bmod 5$$

And,

$$g(x) = \left(x^2 + 3x + 1\right) \bmod 5$$

Consider $J_5 = \{0,1,2,3,4\}$

To prove $f = g$,

Step 2 of 2

To compute $f(x)$ and $g(x)$ for all $x \in J_5$, as shown below:

x	$f(x)=(x+4)^2 \bmod 5$	$g(x)=(x^2+3x+1) \bmod 5$
0	$f(0)=(4)^2 \bmod 5$ $=16 \bmod 5$ $=1$	$g(0)=(1) \bmod 5$ $=1$
1	$f(1)=(1+4)^2 \bmod 5$ $=0$	$g(1)=(1+3+1) \bmod 5$ $=0$
2	$f(2)=(2+4)^2 \bmod 5$ $=36 \bmod 5$ $=1$	$g(2)=(4+6+1) \bmod 5$ $=11 \bmod 5$ $=1$
3	$f(3)=(3+4)^2 \bmod 5$ $=49 \bmod 5$ $=4$	$g(3)=(9+9+1) \bmod 5$ $=19 \bmod 5$ $=4$
4	$f(4)=(4+4)^2$ $=64 \bmod 5$ $=4$	$g(4)=(16+12+1) \bmod 5$ $=29 \bmod 5$ $=4$

The above table of values shows that $f(x)=g(x)$, for all x in J_5.

Hence proved