5.2 Logarithmic Functions and Their Properties

Monday, January 17, 2022 3:53 PM

The logarithmic function is defined by

$$y = f(x) = \log_a x = \frac{\ln x}{\ln a} \quad , \quad x > 0 \quad , \quad a \neq 1$$

$$\Downarrow$$

$$\boxed{\ln 1 = 0}$$

$$y = \log_a x \iff a^y = x$$

logarithmic form $\qquad$ exponential form

Exp write the following logarithmic equation using exponential form

y, x, a

① $4 = \log_2 16 \iff 2^{\boxed{4}} = 16$

② $\frac{1}{2} = \log_9 3 \iff 9^{\frac{1}{2}} = 3$

$\sqrt{9}$

③ $-2 = \log_5 \left(\frac{1}{25}\right) \iff 5^{-2} = \frac{1}{25}$

$\frac{1}{5^2}$

y

$\frac{1}{5^2}$

Exp write the following equations in logarithmic form

① $2^{4} = 16 \iff y = \log_a x$

$4 \to y$, $2 \to a$, $16 \to x$

$4 = \log_2 16$

$2^4 = 16$

② $3^{-1} = \frac{1}{3} \iff y = \log_a x$

$-1 \to y$, $3 \to a$, $\frac{1}{3} \to x$

$-1 = \log_3 \left(\frac{1}{3}\right)$

$3^{-1} = \frac{1}{3}$

Remark ① Natural logarithmic function has the form

$$y = \log_{\boxed{e}} x = \frac{\ln x}{\ln e} = \ln x$$

$\ln e \to 1$

$a = e \approx 2.718$

② Graph $y = \ln x$ ← موجبة , $x > 0$

x	$y = \ln x$

$D = (0, \infty)$

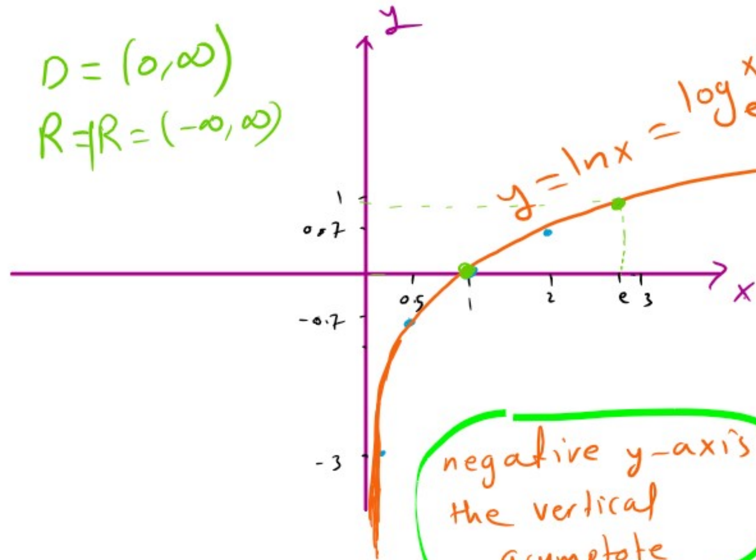
y

x

x	$y=\ln x$
0.05	$\ln 0.05 = -3$
0.1	$\ln 0.1 = -2.3$
0.5	$\ln 0.5 = -0.7$
1	$\ln 1 = 0$
2	$\ln 2 = 0.7$
e	$\ln e = 1$
3	$\ln 3 = 1.1$
10	$\ln 10 = 2.3$

$D=(0,\infty)$

$R=\mathbb{R}=(-\infty,\infty)$



negative y-axis is the vertical asymptote

Using Calculator

$\log x = \log_{10} x$

$= \dfrac{\ln x}{\ln 10}$

Exp $\log 2 \approx 0.301$ ($\log_{10} 2$)

$\ln 2 \approx 0.693$ ($\log_e 2$)

$\ln x = \log_e x$

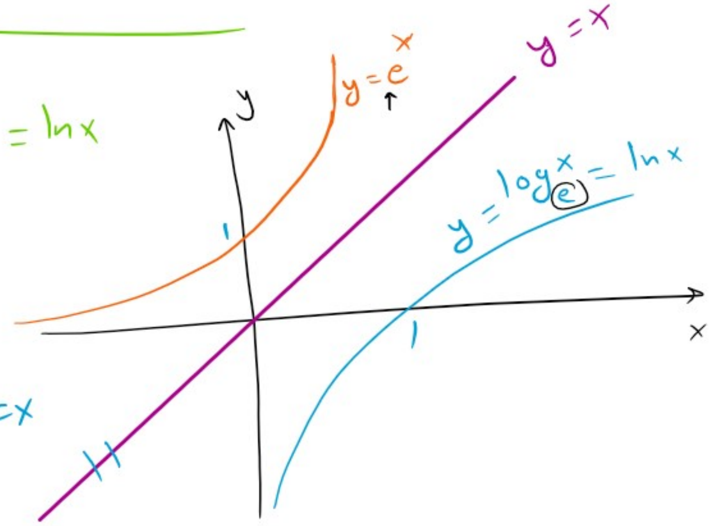
$= \dfrac{\ln x}{\ln e} \to 1$

$= \ln x$

e^x is the inverse of $\log_e x = \ln x$

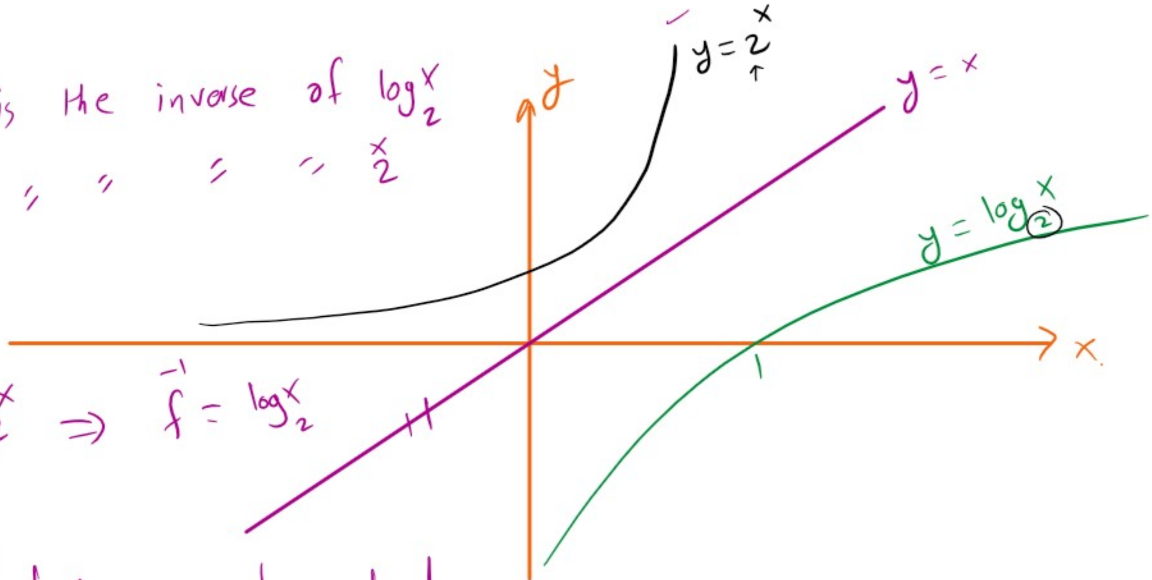
$\ln x$ 〃 〃 〃 〃 e^x

f and f^{-1} are symmetric about $y=x$



2^x is the inverse of $\log_2 x$

$\log_2 x$ 〃 〃 〃 〃 2^x



$f = 2^x \Rightarrow f^{-1} = \log_2 x$

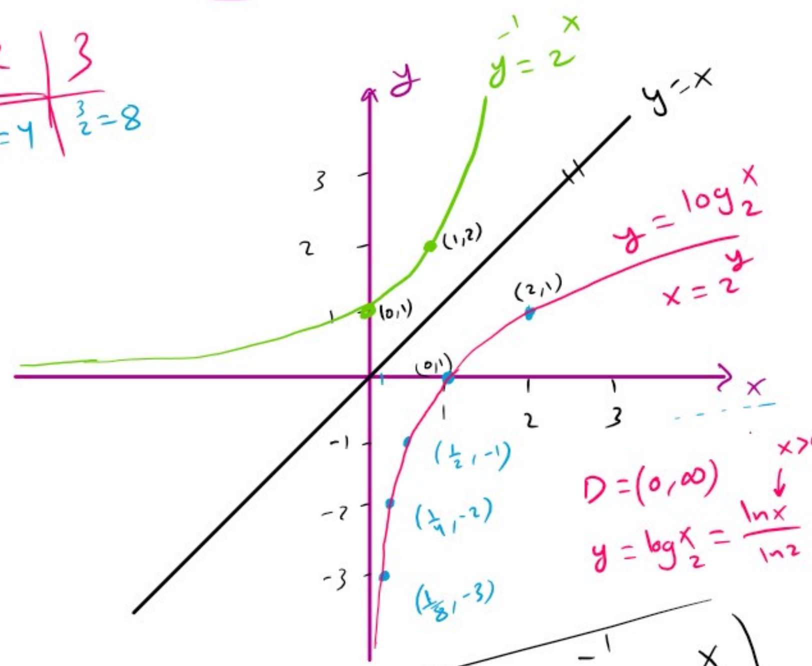
2^x and $\log_2 x$ symmetric about $y = x$

$y = \log_2 x = \dfrac{\ln x}{\ln 2}$ ← أسهل

Exp ① Graph $y = \log_2 x$

$2^y = x$ ← أسهل

y	-3	-2	-1	0	1	2	3
x	$2^{-3} = \frac{1}{8}$	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	1	2	$2^2 = 4$	$2^3 = 8$



② Find the inverse of $\log_2 x$

$y^{-1} = 2^x$

$D = (0, \infty)$

$x > 0$

$y = \log_2 x = \dfrac{\ln x}{\ln 2}$

$$\boxed{f = \log_a x \Rightarrow f^{-1} = a^x}$$

$R = \mathbb{R} = (-\infty, \infty)$

Exp: Solve for x

① $\log_3 x = 4 \Rightarrow 3^4 = x \Rightarrow x = \underbrace{3\cdot3}_{9}\cdot\underbrace{3\cdot3}_{9} = 81$

② $\log_{16} x = -\frac{1}{2} \Rightarrow 16^{-\frac{1}{2}} = x$

$\frac{1}{16^{\frac{1}{2}}} = x$

$\frac{1}{\sqrt{16}} = x$

$\boxed{\frac{1}{4} = x}$

③ $\log(4x+20) = 2$

$\log_{10}(4x+20) = 2$

$10^2 = 4x+20$

$100 = 4x + \cancel{20}$

$-20 \qquad \cancel{-20}$

$\frac{80}{4} = \frac{\cancel{4}x}{\cancel{4}}$

$\boxed{20 = x}$

④ $\ln(3x-17) = 0$

$\ln 1 = 0$

$\Downarrow$

$3x - \cancel{17} = 1$

$\cancel{+17} \quad +17$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{18}{3}$

$\boxed{x = 6}$

$\frac{\ln(3x-17)}{\ln e} = 0$

$1 \rightarrow \ln e$

$\log_e 3x-17 = 0$

$e^0 = 3x-17$

$1 = 3x - \cancel{17}$

$+17 \quad \cancel{+17}$

$18 = 3x$

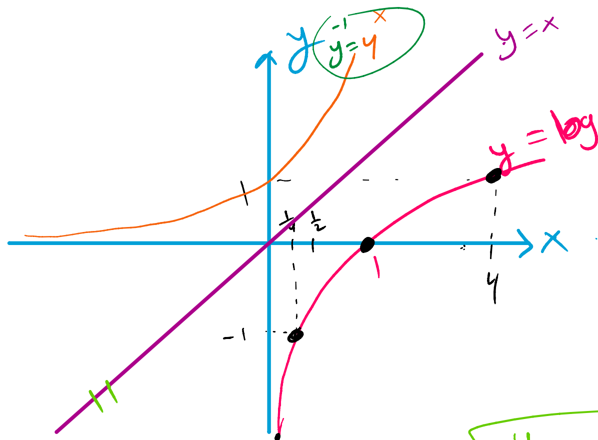
$\boxed{20}$

$\frac{18}{3} = \frac{3x}{3}$

$\boxed{6 = x}$

Exp Graph

① $y = \log_4 x$



x	$4^{-2} = \frac{1}{16}$	$4^{-1} = \frac{1}{4}$	$4^0 = 1$	$4^1 = 4$	$4^2 = 16$
y	-2	-1	0	1	2

$x = 4^y$

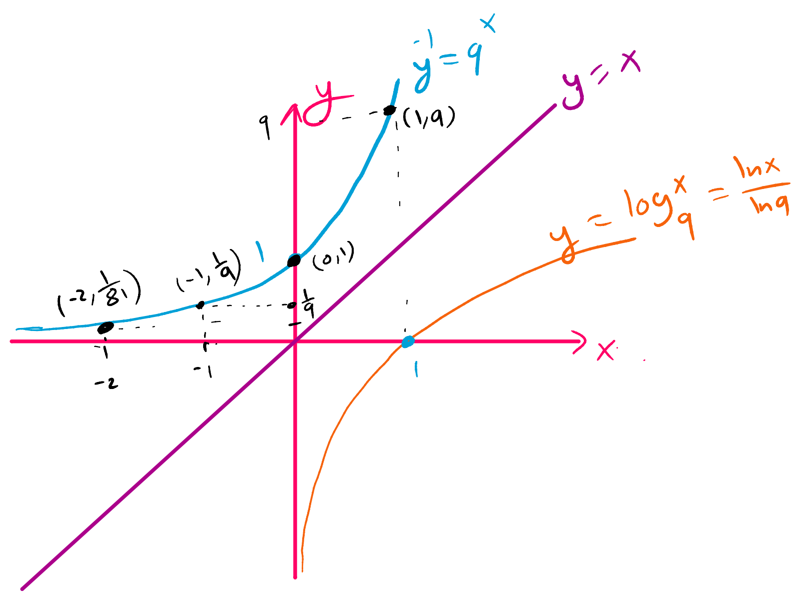
$y = \log_4 x = \frac{\ln x}{\ln 4}$ or $\boxed{4^y = x}$

② $y = 4^x$

4^x is the inverse function of $\log_4 x$

$\log_4 x$ " " " " 4^x

③ $y = \log_9 x$



④ $y = 9^x$

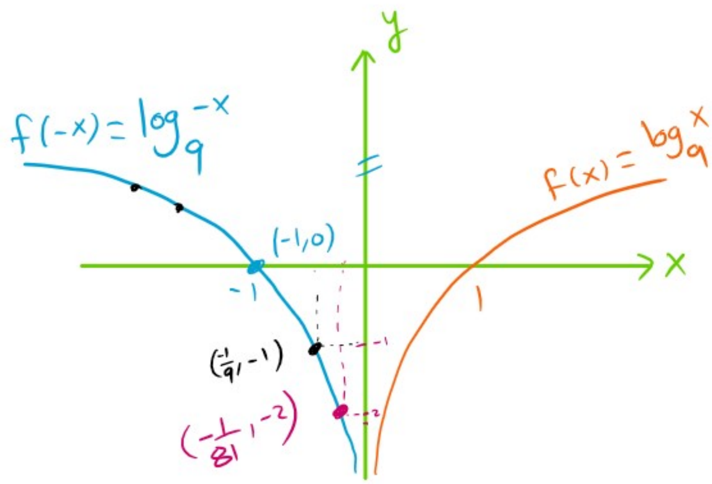
9^x is the inverse function of $\log_9 x$

x	-2	-1	0	1	2
y	$9^{-2} = \frac{1}{81}$	$9^{-1} = \frac{1}{9}$	$9^{0} = 1$	$9^{1} = 9$	$9^{2} = 81$

(5) $y = \log_9 -x$

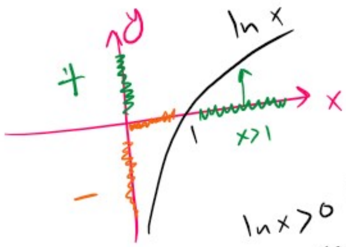
compare with $f(x) = \log_9 x$

$f(-x) = \log_9 -x$



$y = \log_9 -x \Rightarrow 9^y = -x \Rightarrow \boxed{x = -9^y}$

y	-2	-1	0	1	2
x	$-9^{-2} = \frac{-1}{81}$	$-9^{-1} = -\frac{1}{9}$	$-9^{0} = -1$	$-9^{1} = -9$	$-9^{2} = -81$

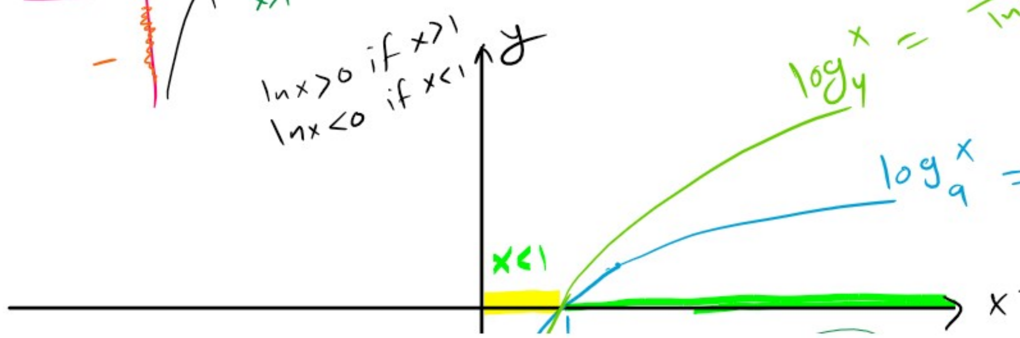


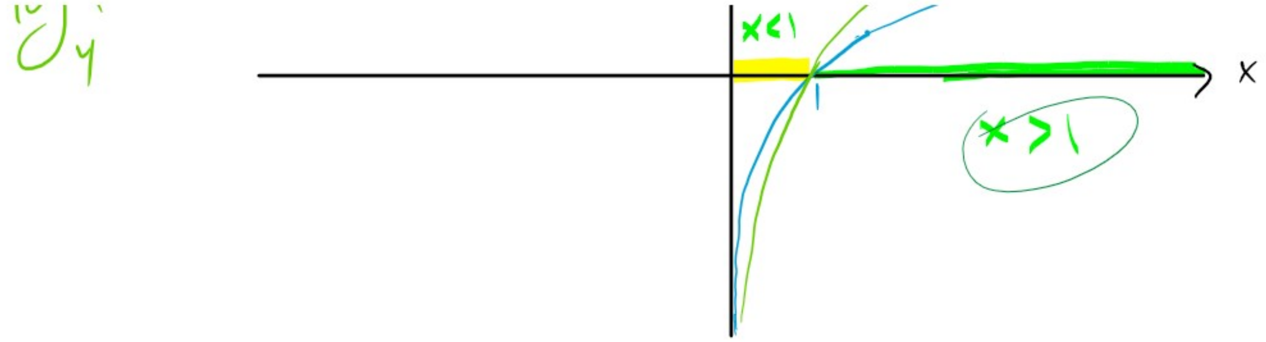
$\log_9 x$

$\log_y x$

$\log_y x = \frac{\ln x}{\ln y}$

$\log_9 x = \frac{\ln x}{\ln 9}$




x<1
x>1
1
x

