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1) $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \ |x|<1$

$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n, \ |x|<1$

2) $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

3) $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

4) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

5) $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$

6) $\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}, \ |x|\le 1$

Q10 Subsitution

$$\frac{1}{2-x} = \frac{1}{2}\left(\frac{1}{1-\frac{x}{2}}\right) = \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n$$

Q12 $x^2 \sin x$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$x^2 \sin x = x^2\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right]$$

$$= x^3 - \frac{x^5}{3!} + \frac{x^7}{5!} - \frac{x^9}{9!} + \cdots$$

متطابقة

Q18 $\sin^2 x = \boxed{\frac{1-\cos 2x}{2}} = \frac{1}{2} - \frac{1}{2}\cos 2x$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x = \frac{1}{2} - \frac{1}{2}\left(1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \cdots\right)$$

$$= \frac{1}{2} - \left(\frac{1}{2} - \frac{(2x)^2}{4} + \frac{(2x)^4}{2(4!)} - \frac{(2x)^6}{2(6!)} + \cdots\right)$$

$$= \frac{(2x)^2}{4} - \frac{(2x)^4}{2(4!)} + \frac{(2x)^6}{2(6!)} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2x)^{2n}}{2(2n)!}$$

Q28 $\ln(1+x) - \ln(1-x)$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$$

$$\ln(1+x) - \ln(1-x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n} - \sum \frac{(-1)^{n-1}(-x)^n}{n}$$

$$= \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots\right) - \left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \cdots\right)$$

$$= 2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \frac{2x^7}{7} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{2x^{2n+1}}{2n+1}$$

Q35 Estimate the error if $P_3(x) = x - \frac{x^3}{6}$ is used to estimate the value of $\sin x$ at $x = 0.1$

$f(x) = P_3(x) + R_3(x)$, where $R_3(x) = \frac{f^{4}(c)(x-a)^4}{4!} = \frac{f^{(4)}(c)\, x^4}{4!}$

نفس ذ واحد الاشتقاق P

remainder او error نفسها

$\text{Error} = R_n(x) \le \frac{M}{(n+1)!}|x-a|^{n+1}$ نفسها

دال M هي الupper bound يعني نساوي 1 لانه ال $-1 \le \sin \le 1$

$R_3(x) \le 1 \frac{|x^4|}{4!}$ دال a=0 (Maclurrine) فنبدل كالآتي ⇐

$R_3(x) \le \frac{(0.1)^4}{24}$ هو طالب بالسؤال $x = 0.1$

$R_3(x) \le 4.2 \times 10^{-6}$

يوقف بس لعند مرحلة الحد اللي هو يحصل الى معطاني اياه بالسؤال

ويبقى الحد اللي بعد R_n هو R_n

Q37 $\sin x = \boxed{x - \frac{x^3}{3!}} + \boxed{\frac{x^5}{5!}}$ نفسها هاي

$\text{error}: R_n \le \frac{M|x-a|^{n+1}}{(n+1)!}$ a=0 (Maclurine) و M = 1 لانه $|\sin| \le 1$

$\text{error} \le \frac{|x^5|}{5!}$ → هو حاكيلي بالسؤال انه الايرور لا يتجاوز 5×10^{-4}

نبدي لحتى هاي القيمة أقل من 5×10^{-4} عشان اوجد قيمة

$$\frac{|x^5|}{5!} < 5\times10^{-4} \Rightarrow \frac{|x^5|}{120} < 5\times10^{-4} \Rightarrow |x|^5 < 600\times10^{-4} \Rightarrow |x| < \sqrt[5]{600\times10^{-4}} \Rightarrow x \approx 0.56968$$

Q41 $e^x = 1 + x + \frac{x^2}{2}$, when x is small

$f(x) = P_n(x) + R_n(x)$

$= 1 + x + \frac{x^2}{2} + R_n(x)$

$= 1 + x + \frac{x^2}{2} + \frac{e^c}{(n+1)!}x^{n+1}$

$\text{error} = R_n(x) < \frac{M|x-a|^{n+1}}{(n+1)!}$

$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}$

$f = e^x$

$f' = e^x$

$f'' = e^x$

$\vdots$

$f^{(n+1)}(c) = e^c \quad \therefore R_n(x) = \frac{e^c}{(n+1)!}x^{n+1}$