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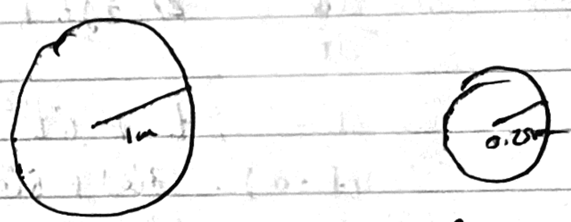
Principles of Physics (141) 10th edition
Discussion
Chapter 10: Rotation

Problems: 1, 4, 12, 25, 34, 45, 52, 66

Problem 1: A tractor has rear (عجلات خلفية) wheels with a radius of 1.00 m and front (عجلات أمامية) wheels with a radius of 0.250 m. The rear wheels are rotating at 100 rev/min. Find ⓐ the angular speed of the front wheels in revolutions per minute and ⓑ the distance covered by the tractor in 10.0 min.

sol: a) $v_f = v_r$



rear Wheel
$\omega_r = 100$ rev/min

front wheel
$\omega_f = ?$

$$r_f \omega_f = r_r \omega_r$$

$$\omega_f = \frac{r_r}{r_f}\,\omega_r$$

$$= \frac{1}{0.25} \times 100 \,\frac{\text{rev}}{\text{min}}$$

$$\boxed{\omega_f = 400 \text{ rev/min}}$$

b) The distance that tractor passes in 10 min is the distance passed in one rotation ($2\pi r$) multiplied by the number of rotation

$d = \cancel{\quad}$

$$\Rightarrow \; d = 2\pi r\,(\omega_r t)$$

$$= 2\pi(1)\left(100\,\frac{\text{rev}}{\text{min}} \times 10 \text{ min}\right)$$

$$= 2\pi \text{m}\,(1000 \text{ rev})$$

عدد اللفات 1000 في 10 دقائق

$$= 2000\pi \text{ m}$$

$$= 6280 \text{ m}$$

$$= 6.28 \times 10^3 \text{ m in } 10 \text{ min}$$

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Problem 4: The angular position of a point on a rotating wheel is given by $\theta = 2.0 + 4.0t^2 + 2.0t^3$, Where θ is in radians and t is in seconds. At $t=0$, what are (a) the point's angular position and (b) its angular velocity? (c) What is its angular velocity at $t = 3.0$ sec? (d) calculate its angular acceleration at $t = 4.0$ sec. (e) Is ~~this~~ its angular acceleration constant?

sol

a) $\theta(t=0) = 2 + 4(0)^2 + 2(0)^3 = 2$ radians

b) $w = \dfrac{d\theta}{dt} = 4(2)t + 2(3)t^2$

$w = 8t + 6t^2$

$w(t=0) = 8(0) + 6(0)^2 = 0$ rad/sec

c) $w(t=3) = 8(3) + 6(3)^2$

$= 24 + 6(9)$

$= 24 + 54$

$W = 78$ rad/sec

d) $\alpha = \dfrac{dw}{dt} = \dfrac{d^2\theta}{dt^2} = 8 + 12t$

$\alpha(t=4) = 8 + 12(4)$

$\boxed{\alpha = 98 \text{ rad/sec}^2}$

e) No because $\alpha = 8 + 12t$, t variable.

Problem 12: The anguler speed of automobile engine is increased at a constant rate from 1200 rev/min to 3200 rev/min in 12 sec. (a) What is its angular acceleration in revolutions per minute - squared? (b) How many revolutions does the engine make during this 12 sec interval?

Sol:

$\omega_i = 1200$ rev/min

$\omega_f = 3200$ rev/min

$t = 12$ sec $= 12 \text{ sec} \times \frac{1 \text{ min}}{60 \text{ sec}} = 0.2$ min

a) ~~wp~~: increased at constant rate (constant acceleration)

$$\omega_f = \omega_i + \alpha t$$

$$3200 = 1200 + \alpha(0.2)$$

$$3200 - 1200 = 0.2\alpha$$

$$2000 = 0.2\alpha$$

$$\frac{2000}{0.2} = \alpha \Rightarrow \alpha = 10000 \text{ rev/min}^2$$

$$= 10 \times 10^3 \text{ rev/min}^2$$

b) $$\theta_f - \theta_i = \frac{1}{2}(\omega_i + \omega_f)t$$

$$\theta = \frac{1}{2}(1200 + 3200) \times 0.2$$

$$= \frac{1}{2}(4400) \times 0.2$$

$$= 440 \text{ ~~revilons~~ revolutions}$$

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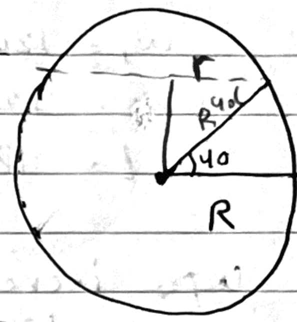
Problem 25: What is the angular speed ω about the polar axis of a point on earth's surface at latitude (خط عرض) $40°N$ (شمال)? (Earth rotates about that axis.) (b) What is the linear speed v of the point? What are (c) ω and (d) v for a point at equator?

sol: The linear speed of a point on the earth's surface depends on its distance from the axis of rotation.

For linear speed $\boxed{v = \omega r}$

Wher r is the radius of orbit



$\cos 40 = \frac{r}{R}$

$\Rightarrow r = R\cos 40$

A point on earth at latitude of $40°$ moves along a circular path of radius

$\boxed{r = R\cos 40}$

Wher R is the earth's radius and $\boxed{R = 6.38 \times 10^6 \ m}$

* on the other hand $r = R$ at equator على خط الاستواء

* the earth makes one rotation per day

$1d = 24\,h = 24\,h \times 60 \frac{min}{h} \times 60 \frac{sec}{min} = 86400$

$\Rightarrow 1d = 8.64 \times 10^2 \ sec$

a) So the angular speed of earth is given by

$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{8.64 \times 10^2} = 7.26 \times 10^{-5} \simeq 7.3 \times 10^{-5} \ rad/s$

b) At latitude of $40°$ the linear speed v

$v = \omega r = \omega R\cos\theta = 7.26 \times 10^{-5} \times 6.38 \times 10^6 \times \cos 40$

$= 3.5 \times 10^2 \ m/s$

c) The value of angular speed (W) is the same for all points on the earth, Hence at equator:

$W = 7.3 \times 10^{-5}$ rad/s

d) The latitude at equator is $0°$, Hence the speed

$v = Wr = WR\cos 0 = WR$

$\Rightarrow v = 7.26 \times 10^{-5} \times 6.38 \times 10^{6} = 4.6 \times 10^{2}$ m/s.

Problem 34: Figure 10-23 gives angular speed versus time for a thin rod that rotates around one end. The scale on the W axis is set by $W_s = 6.0$ rad/sec. a) What is the magnitude of the rod's angular acceleration? b) At $t = 4.0$ sec the rod has a rotational kinetic energy of 1.60 J. What is its kinetic energy at $t = 0$?

sol:

a) $\alpha = \frac{dW}{dt}$ slope

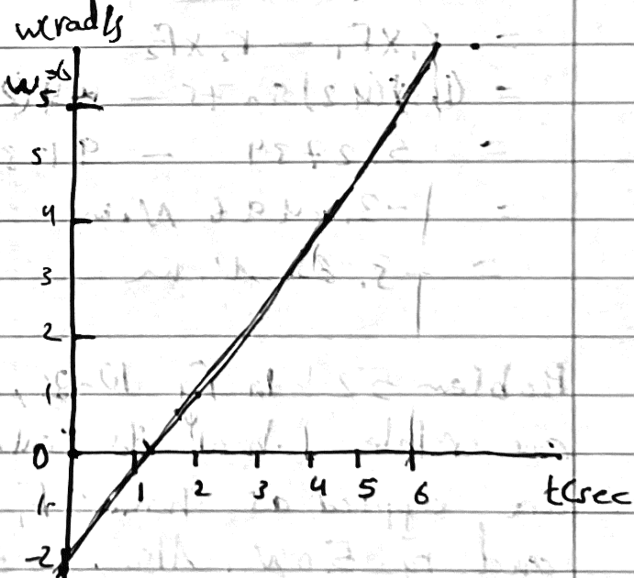
slop $= \frac{\Delta W}{\Delta t}$ straight line

$\Rightarrow \alpha = \frac{\Delta W}{\Delta t} = \frac{W(4) - W(0)}{4 - 0}$

$= \frac{4 - (-2)}{4}$

$= \frac{6}{4}$

$= 1.5$ rad/sec



b) $k = \frac{1}{2} I W^2 \Rightarrow k(t=4) = \frac{1}{2} I W(4)^2$

$I = \frac{2k}{W(t=4)^2} = \frac{2 \times 1.6}{(4)^2} = \frac{3.2}{16} = 0.2$ kg.m²

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$k(t=0) = \frac{1}{2} I (\omega(t=0))^2$

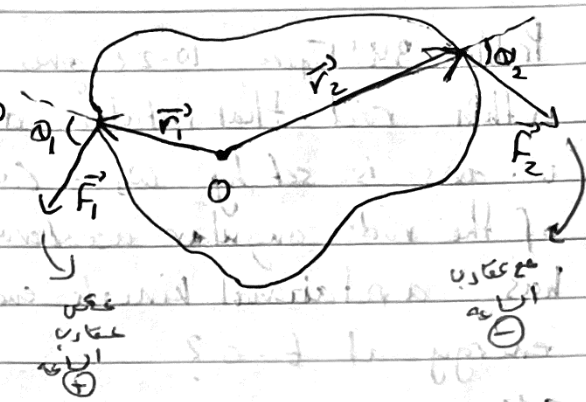
$= \frac{1}{2} \times 0.2 \times (-2)^2$

$= 0.4 \, J$

Problem 45: The body in Fig 10-30 is pivoted at O, and two forces acts on it as shown. If $r_1 = 1.3 m$, $r_2 = 2.15 m$, $F_1 = 4.20 N$, $F_2 = 4.9 N$, $\theta_1 = 75.0°$ and $\theta_2 = 60.0°$, what is the net torque about the pivot?

Sol: $F_1 = 4.20 N$, $r_1 = 1.3 m$, $\theta_1 = 75.0$

$F_2 = 4.90 N$, $r_2 = 2.15 m$, $\theta_2 = 60.0$



$\tau_{net} = \vec{\tau_1} + \vec{\tau_2}$

$\cancel{= F_1 \times r_1 - F_2 \times r_2}$

$= r_1 \times F_1 - r_2 \times F_2$

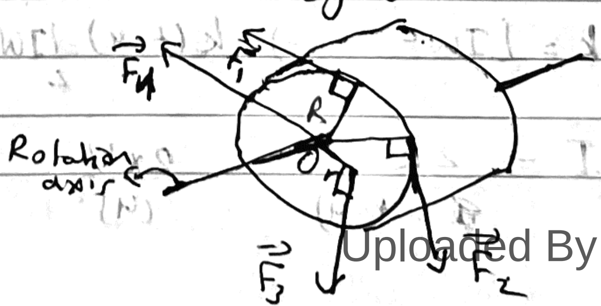
$= (1.3)(4.2) \sin 75 - (2.15)(4.90) \sin 60$

$= 5.2739 - 9.1235$

$= -3.8496 \, N.m$

$= -3.85 \, N.m$

Problem 52: In fig 10-33, a cylinder having a mass of 3.0 kg can rotate about its central axis through point O. Forces are applied as shown: $F_1 = 6.0 N$, $F_2 = 4.0 N$, $F_3 = 2.0 N$ and $F_4 = 5.0 N$. Also, $r = 5.0$ cm and $R = 12$ cm. Find the a) magnitude and b) direction of the angular acceleration of the cylinder. (During the rotation, the forces maintain their same angles relative to the cylinder.



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sol : $m = 3\,kg$ بين r_1 و F_1

$F_1 = 6.0N$, $\theta_1 = 90$, $r_1 = R = 12\,cm = 12\times10^{-2}\,m$

$F_2 = 4.0N$, $\theta_2 = 90$, $r_2 = R = 12\times10^{-2}\,m$

$F_3 = 2.0N$, $\theta_3 = 90$, $r_3 = r = 5\,cm = 5\times10^{-2}$

$F_4 = 5.0N$, $\theta_4 = zero$, $r_4 = R = 12\times10^{-2}\,m$

no rotation due to F_4 (no torque $\tau_4 = r_4 F_4 \sin\theta = zero$)

$$\tau_{net} = \underbrace{\vec{\tau}_1}_{\text{عكس عقارب الساعة}} + \underbrace{\vec{\tau}_2}_{\text{مع عقارب الساعة}} + \underbrace{\vec{\tau}_3}_{\text{مع عقارب الساعة}} + \cancel{\vec{\tau}_4}^{\,zero}$$

(+) عكس عقارب الساعة

(−) مع عقارب الساعة

$$\tau_{net} = RF_1\sin90 - RF_2\sin90 - rF_3\sin90$$

$$\tau_{net} = RF_1 - RF_2 - rF_3$$

$$\tau_{net} = R(F_1 - F_2) - rF_3$$

$$\tau_{net} = 12\times10^{-2}(6.0 - 4.0) - 5\times10^{-2}(2.0)$$

$$\tau_{net} = 12\times10^{-2}\times 2 - 5\times10^{-2}(2.0)$$

$$\boxed{\tau_{net} = 14\times10^{-2}\ N.m}$$ counterclock wise

⇒ moment of inertia $I = \frac{1}{2}mR^2$

$$= \frac{1}{2}(3)(12\times10^{-2})^2$$

$$= 216\times10^{-4}$$

$$\Rightarrow \alpha = \frac{\vec{\tau}_{net}}{I} = \frac{14\times10^{-2}}{216\times10^{-4}}$$

$= 6.48\ rad/sec^2$ counterclock wise

$\tau \Rightarrow F$

$I \Rightarrow m$

$\alpha \Rightarrow a$

Problem 66: A uniform spherical shell of mass $M = 4.5\,kg$ and radius $R = 8.5\,cm$ can rotate about a vertical axis on frictionless bearings (Fig 10-38). A massless cord passes around the equator of the shell, over a pulley of rotational inertia $I = 3.0\times10^{-3}\,kg.m^2$ and radius $r = 5.0\,cm$, and is attached to a small object of

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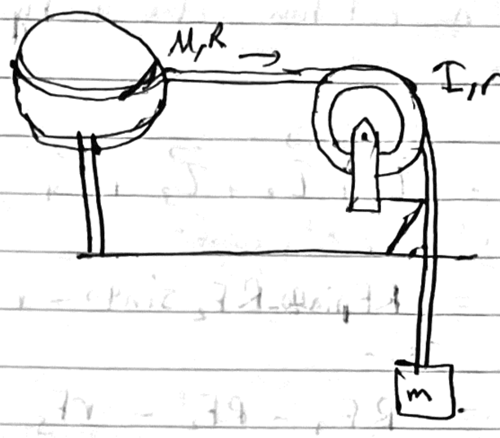
mass $m = 0.60\ kg$. There is no friction on the pulley's axle; the cord doesn't slip on the pulley. What is the speed of the object when it has fallen 82 cm after being released from rest? Use energy considerations.

$M = 4.5\ kg$
$m = 0.60\ kg$
$R = 8.5\ cm = 8.5\times10^{-2}\ m$
$r = 5\ cm = 5\times10^{-2}\ m$
$I = 3\times10^{-3}\ kg.m^2$
$h = 82\ cm = 0.82\ m$



sol

$$\Delta K = W$$
$$K_f - K_i = W$$
$$K_f = W$$
$$\frac{1}{2}mv^2 + \frac{1}{2}I_{shell}\,\omega_{shell}^2 + \frac{1}{2}I_{pulley}\,\omega_{pulley}^2 = mgh$$
$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}MR^2\right)\left(\frac{v}{R}\right)^2 + \frac{1}{2}I_p\left(\frac{v}{r}\right)^2 = mgh$$
$$mv^2 + \left(\frac{2}{3}M\cancel{R^2}\right)\frac{v^2}{\cancel{R^2}} + I_p\frac{v^2}{r^2} = 2mgh$$
$$v^2\left(m + \frac{2}{3}M + \frac{I_p}{r^2}\right) = 2mgh$$
$$v^2 = \frac{2mgh}{\left(m + \frac{2}{3}M + \frac{I_p}{r^2}\right)}$$
$$v = \sqrt{\frac{2mgh}{\left(m + \frac{2}{3}M + \frac{I_p}{r^2}\right)}} = \sqrt{\frac{2\times0.60\times9.8\times82\times10^{-2}}{\left(0.6 + \frac{2}{3}(4.5) + \frac{3\times10^{-3}}{(5\times10^{-2})^2}\right)}}$$
$$= \sqrt{2.009} = 1.41\ m/s$$

