9.3 Hypothesis Testing about the Population Mean (μ) when σ is known

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Test statistics: A statistic whose value helps to determine whether H_0 should be rejected.

For example: the test statistic for hypothesis tests about population mean when σ is known is

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

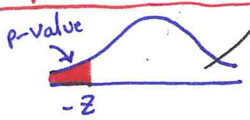
There are two approaches in Hypothesis Testing:

1. p-value approach: uses the value of the test statistic z to compute p-value.

   p-value: is the prob. that provides a measure of the evidence against H_0 provided by the sample. "Smaller p-values indicate more evidence against H_0".

2. Critical value approach: uses a critical value (z_α or $z_{\alpha/2}$) to compare with the test statistic z in order to determine wheather H_0 should be rejected.

	Lower Tail Test	Upper Tail Test	Two Tailed Test
Hypotheses	$H_0: \mu \geq \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$
Test statistic	$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$	$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$	$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$
Rejection Rule using p-value approach	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$ (p-value, z)	Reject H_0 if p-value $\leq \alpha$ ($\frac{p}{2}$, $\frac{p}{2}$, $-z$, z)
Rejection Rule using Critical value approach	Reject H_0 if $z \leq -z_\alpha$ (α, $-z_\alpha$)	Reject H_0 if $z \geq z_\alpha$ (α, z_α)	Reject H_0 if $z \leq -z_{\alpha/2}$ or $z \geq z_{\alpha/2}$ ($\alpha/2$, $\alpha/2$, $-z_{\alpha/2}$, $z_{\alpha/2}$)



If the confidence interval $\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ contains μ_0, do not reject H_0. Otherwise, reject H_0.

Example (Q9 page 350)

Consider the following hypothesis test:

$H_0: \mu \geq 20$

$H_a: \mu < 20$

lower Tail Test

A sample of 50 provided a sample mean of 19.4

The population standard deviation is 2

[a] Compute the value of the test statistic?

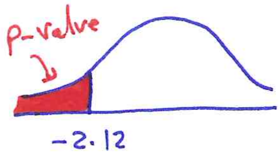
$n = 50$, $\sigma = 2$

$\bar{x} = 19.4$, $\mu_0 = 20$

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{19.4 - 20}{\frac{2}{\sqrt{50}}} = \frac{-0.6}{0.283} = -2.12$$

[b] what is the p-value?

From the standard normal table, we have



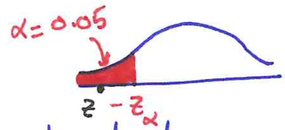
p-value = 0.0170

[c] Using $\alpha = 0.05$, what is your conclusion?

Reject H_0 since p-value $= 0.0170 \leq \alpha = 0.05$

[d] what is the rejection rule using the critical value? what is your conclusion?

Reject H_0 if $z \leq -z_\alpha = -1.645$



From the standard normal table

$\Rightarrow z_\alpha = -1.645$

since $-2.12 \leq -1.645$, we reject H_0

(Q 10 page 351) Consider the following hypothesis test

$H_0: \mu \leq 25$

$H_a: \mu > 25$

Upper Tail Test

A sample of 40 provided a sample mean of 26.4.

The population standard deviation is 6.

[a] Compute the value of the test statistic.

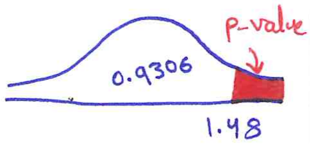
$n = 40$, $\sigma = 6$

$\bar{x} = 26.4$, $\mu_0 = 25$

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{26.4 - 25}{\frac{6}{\sqrt{40}}} = \frac{1.4}{0.949} = 1.48$$

[b] what is the p-value?

From the standard normal table, we have



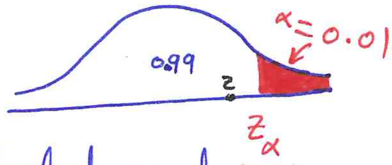
p-value $= 1 - 0.9306 = 0.0694$

[c] At $\alpha = 0.01$, what is your conclusion?

Do not reject H_0 since p-value $> \alpha$ i.e. $0.0694 > 0.01$

[d] what is the rejection rule using the critical value? what is your conclusion?

Reject H_0 if $z \geq z_\alpha = 2.33$



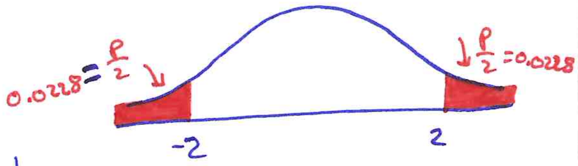
since $1.48 < 2.33$, do not reject H_0 | from the standard normal table, we have $z_\alpha = z_{0.01} = 2.33$

Example (Q11 page 351) Consider the following hypothesis test $H_0: \mu = 15$, $H_a: \mu \neq 15$

A sample of 50 provided a sample mean of 14.15. The population standard deviation is 3. Two Tailed Test

[a] Compute the value of the test statistic $n = 50$, $\sigma = 3$, $\bar{x} = 14.15$, $\mu_0 = 15$

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{14.15 - 15}{\frac{3}{\sqrt{50}}} = -2$$



[b] Compute the p-value?

From the standard normal table, we have

p-value $= 0.0228 + 0.0228 = 0.0456$

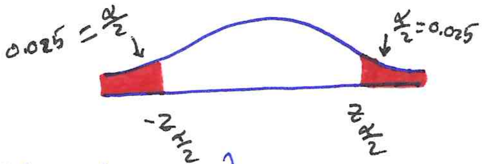
[c] At $\alpha = 0.05$, what is your conclusion?

Reject H_0 since p-value $= 0.0456 \leq \alpha = 0.05$.

[d] what is the rejection rule using the critical value? what is your conclusion.

Reject H_0 if $z \leq -z_{\alpha/2} = -1.96$

or if $z \geq z_{\alpha/2} = 1.96$



since $-2 \leq -1.96$, we reject H_0 | From the standard normal table, we have $-z_{\alpha/2} = -1.96$

Notes:

- If the sample size $n \geq 30$, then we can use hypothesis tests above
- If the sample size $n < 30$ and the population is normally distribution, then we can use the hypothesis tests above. (exact)
- If the sample size $n < 30$ and " " " not " " but is symmetric, then sample size as small as 15 is good to be able to provid acceptable results.